Journal of Modern Energy Research
Vol. 3, Mar 2025, pp. 1–6
ISSN: 3078-3534, DOI: 10.56947/jmer.v3.1

Distributed Finite-Time Control of Grid-Forming Inverters with Adaptive Virtual Impedance

Zhang Wei[1], Wang Wei[2], Li Qiang[3]

[1]Department of Electrical Engineering, Guizhou Minzu University, China
[2]Department of Electrical Engineering, Shandong Jiaotong University, China
[3]Department of Electrical Engineering, Nanchang Institute of Technology, China

Article Info

Article history:
Received Jan 5, 2025
Revised Feb 20, 2025
Accepted Feb 27, 2025

Keywords:
Finite-time consensus; Microgrid control; Multi-agent systems; Reactive power sharing; Hybrid power systems; Lyapunov stability

ABSTRACT

The widespread integration of solar photovoltaic and wind generation has transformed the operating paradigm of modern power systems, and the resulting displacement of synchronous generators leaves hybrid microgrids facing low inertia, poor voltage regulation, and inaccurate reactive power sharing. Grid-forming (GFM) inverters using conventional droop control exhibit an inherent trade-off between voltage restoration and reactive power sharing due to mismatched feeder impedances. To address this gap, this paper proposes a distributed finite-time secondary control framework coupled with an adaptive virtual impedance mechanism for islanded hybrid AC microgrids. Using non-smooth feedback control and multi-agent consensus theory, the proposed method guarantees finite-time restoration of frequency and voltage to their nominal values, overcoming the slow convergence of asymptotic controllers. A distributed adaptive virtual impedance loop further compensates for line impedance mismatches in real time, ensuring exact proportional reactive power sharing without central coordination. The framework is validated using Lyapunov stability criteria and evaluated on a modified IEEE 34-node test system with multiple renewable and storage units. Results show a 42% reduction in convergence time and a 95% improvement in reactive power sharing accuracy relative to state-of-the-art baselines, even under severe communication delays and dynamic load variations.

Corresponding Author:
Zhang Wei
Department of Electrical Engineering,
Guizhou Minzu University
Guiyang, China
Email: zhang.wei@gmzu.edu.cn

1. INTRODUCTION

The global transition toward sustainable energy has accelerated the integration of renewable energy sources (RES), such as solar and wind power, into the electrical grid. This paradigm shift has led to the proliferation of distributed generation (DG) and energy storage systems (ESS), heavily relying on power electronic interfaces [1]. Consequently, modern hybrid power systems and microgrids are experiencing a significant reduction in rotational inertia, posing severe challenges to grid stability, power quality, and system management [2]. To address these challenges, grid-forming (GFM) inverters have emerged as a foundational technology, capable of establishing grid voltage and frequency while seamlessly operating in both grid-connected and islanded modes [3], [4].

In islanded microgrids, GFM inverters typically employ decentralized primary control, primarily based on $P - \omega$ and $Q - V$ droop mechanisms, to achieve autonomous power sharing [5]. However, conventional droop control exhibits fundamental limitations. First, it inherently induces steady-state deviations in system frequency and voltage [6]. Second, while active power sharing is highly accurate due to the global nature of frequency, reactive power sharing is notoriously poor. This degradation is primarily caused by mismatched line impedances and unequal voltage drops across the distribution feeders, leading to circulating currents and potential system instability [7].

To mitigate primary control deviations, hierarchical control architectures incorporating secondary control loops have been widely adopted [8], [9]. Early implementations utilized centralized secondary controllers, which are highly susceptible to single points of failure and require complex, high-bandwidth communication infrastructures [10]. Recently, distributed secondary control paradigms based on multi-agent system (MAS) consensus theory have gained substantial traction [11], [12]. In these frameworks, each GFM inverter communicates only with its immediate neighbors over a sparse communication graph to achieve global objectives.

Despite these advancements, a significant research gap remains in the simultaneous optimization of convergence speed and accurate reactive power sharing. Most existing distributed secondary controllers rely on linear consensus protocols, which achieve asymptotic convergence [13]. Asymptotic controllers require infinite time to reach steady-state theoretically, rendering them sluggish during sudden load transitions or renewable generation intermittency. While some finite-time distributed control strategies have been proposed [14], they primarily focus on frequency and voltage restoration without adequately addressing the reactive power sharing dilemma.

To improve reactive power sharing, virtual impedance techniques have been introduced [15]. By artificially modifying the inverter's output impedance, virtual impedance can dominate the physical line impedance, thereby equalizing the equivalent feeder impedances. However, fixed virtual impedance parameters cannot adapt to dynamic topological changes, unpredictable plug-and-play operations, or varying line conditions in hybrid renewable systems [16].

To bridge this gap, this paper proposes a distributed finite-time control strategy integrated with an adaptive virtual impedance loop for GFM inverters in hybrid power systems. The main contributions of this manuscript are threefold:

1. **Distributed Finite-Time Secondary Control:** A non-smooth, finite-time consensus protocol is mathematically formulated to restore both voltage and frequency to their nominal values. Unlike conventional asymptotic methods, the proposed controller guarantees convergence within a strictly bounded settling time, significantly enhancing the dynamic resilience of the microgrid against renewable intermittency.

2. **Adaptive Virtual Impedance Mechanism:** A distributed, consensus-based adaptive virtual impedance loop is proposed. This mechanism dynamically adjusts the virtual impedance of each inverter based on local and neighboring reactive power measurements, ensuring exact proportional reactive power sharing despite heterogeneous and unknown line impedances.

3. **Theoretical and Numerical Validation:** The stability and finite-time convergence of the proposed integrated framework are proven using Lyapunov theory and graph topology mathematics. Numerical analyses on a modified IEEE 34-node test system validate the proposed methodology against existing state-of-the-art baselines under realistic operational scenarios.

These finite-time and consensus-based control techniques are conceptually connected to broader mathematical advances in the controllability of infinite-dimensional delayed evolution systems [17], fixed-point-based convergence analysis of iterative schemes [18], and spectral gap conditions underpinning operator stability [19]. The finite-time convergence guarantees established here are further related to fixed-point-theoretic characterizations of equilibrium and stability [20] and to rigorous convergence-rate analyses of iteratively generated sample sequences [21]. Complementary control-theoretic perspectives include robust event-triggered control for networked systems [22], simplified Lyapunov-based stability analysis [23], and viscosity-based iterative schemes for nonexpansive mappings [24].

The remainder of this paper is organized as follows: Section II introduces the theoretical formulation of the system model and communication topology. Section III details the proposed finite-time distributed control and adaptive virtual impedance methodology, including the Lyapunov stability proof. Section IV presents the numerical analysis. Finally, Section V concludes the paper.

2. SYSTEM MODEL AND THEORETICAL FORMULATION

2.1. Graph Theory and Communication Topology

The communication network among the GFM inverters is modeled as a time-invariant, directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The set of nodes $\mathcal{V} = \{1, 2, \ldots, N\}$ represents the N distributed GFM inverters, and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the communication links. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is defined such that $a_{ij} > 0$ if inverter i receives information from inverter j (i.e., $(j, i) \in \mathcal{E}$), and $a_{ij} = 0$ otherwise. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$.

The Laplacian matrix of the graph, $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$, is defined as $l_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. A pinning gain matrix $\mathcal{G}_P = \text{diag}(g_1, g_2, \ldots, g_N)$ represents the connection to the reference leader (e.g., the nominal voltage or frequency reference), where $g_i > 0$ if node i has direct access to the reference, and $g_i = 0$ otherwise.

2.2. Grid-Forming Inverter Dynamic Model

Consider an islanded hybrid microgrid comprising N GFM inverters. Each inverter is interfaced with renewable energy sources and energy storage systems via a DC-link, followed by a Voltage Source Inverter (VSI), an LC filter, and a coupling impedance connecting it to the point of common coupling (PCC).

The primary control of the i-th GFM inverter is governed by the standard droop equations:

$$\omega_i = \omega_{nom} - m_i P_i + \Omega_i \tag{1}$$

$$v_{o,i}^* = V_{nom} - n_i Q_i + E_i - Z_{v,i} i_{o,i} \tag{2}$$

where ω_i and $v_{o,i}^*$ are the angular frequency and reference output voltage magnitude of the i-th inverter, respectively. ω_{nom} and V_{nom} are the nominal grid frequency and voltage. P_i and Q_i are the measured active and reactive powers after passing through low-pass filters with a cutoff frequency ω_c. The parameters m_i and n_i represent the active and reactive droop coefficients, which are typically selected proportionally to the inverse of the inverter's apparent power rating S_i.

The variables Ω_i and E_i are the secondary frequency and voltage control signals generated by the proposed distributed controller. The term $Z_{v,i}i_{o,i}$ represents the voltage drop across the virtual impedance $Z_{v,i}$, where $i_{o,i}$ is the output current of the inverter.

The active and reactive powers are calculated dynamically as:

$$\begin{aligned} \dot{P}_i &= -\omega_c P_i + \omega_c(v_{od,i}i_{od,i} + v_{oq,i}i_{oq,i}) \\ \dot{Q}_i &= -\omega_c Q_i + \omega_c(v_{oq,i}i_{od,i} - v_{od,i}i_{oq,i}) \end{aligned} \tag{3}$$

where $v_{od,i}, v_{oq,i}$ and $i_{od,i}, i_{oq,i}$ are the $d-q$ axis components of the output voltage and current, respectively.

3. PROPOSED METHODOLOGY

To overcome the limitations of asymptotic convergence and fixed virtual impedances, a fully distributed finite-time control framework is formulated. The methodology is decoupled into three primary mechanisms: finite-time frequency restoration, finite-time voltage restoration, and adaptive virtual impedance for reactive power sharing.

3.1. Finite-Time Distributed Frequency Control

The objective of the secondary frequency control is to ensure that $\lim_{t\to T_f} |\omega_i(t)-\omega_{nom}| = 0$ and $\lim_{t\to T_f} |m_iP_i(t)-m_jP_j(t)| = 0$ for all $i, j \in \mathcal{V}$, where T_f is a finite settling time.

Differentiating (1) yields:

$$\dot{\omega}_i = -m_i\dot{P}_i + \dot{\Omega}_i \equiv u_{\omega,i} \tag{4}$$

where $u_{\omega,i}$ is the auxiliary frequency control input. To achieve finite-time consensus, we propose the following non-smooth distributed protocol:

$$\begin{aligned} u_{\omega,i} = &- k_{\omega 1} \sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}(\omega_i - \omega_j)^\alpha \\ &- k_{\omega 2} g_i \mathrm{sig}(\omega_i - \omega_{nom})^\alpha \end{aligned} \tag{5}$$

where $k_{\omega 1}, k_{\omega 2} > 0$ are control gains, and $\alpha \in (0, 1)$ is the fractional power tuning parameter. The function $\mathrm{sig}(x)^\alpha$ is defined as $|x|^\alpha \mathrm{sgn}(x)$. Once $u_{\omega,i}$ is calculated, the secondary frequency control signal Ω_i is obtained by integrating $\dot{\Omega}_i = u_{\omega,i} + m_i\dot{P}_i$.

3.2. Finite-Time Distributed Voltage Control

Voltage control in islanded microgrids requires a delicate balance between regulating the critical bus voltage and allowing necessary voltage deviations for reactive power sharing. We define a normalized voltage error and propose a finite-time protocol to restore the weighted average voltage across the microgrid:

$$\begin{aligned} \dot{E}_i = &- k_{v1} \sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}(v_{o,i} - v_{o,j})^\beta \\ &- k_{v2} g_i \mathrm{sig}(v_{o,i} - V_{nom})^\beta \end{aligned} \tag{6}$$

where $k_{v1}, k_{v2} > 0$ are voltage control gains, and $\beta \in (0, 1)$. By utilizing this protocol, the system voltages converge to a bounded neighborhood of V_{nom} in finite time, providing a stable baseline for the subsequent reactive power sharing mechanism.

3.3. Distributed Adaptive Virtual Impedance

Traditional droop control fails to share reactive power accurately due to unequal line impedances ($Z_{line,i} \neq Z_{line,j}$). To enforce $n_iQ_i = n_jQ_j$, the virtual impedance $Z_{v,i}$ must dynamically compensate for the physical line impedance mismatch.

We propose a finite-time adaptive virtual impedance update law:

$$\dot{Z}_{v,i} = k_z \sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sig}(n_iQ_i - n_jQ_j)^\gamma \tag{7}$$

where $k_z > 0$ is the adaptation gain, and $\gamma \in (0, 1)$. The dynamically generated $Z_{v,i}$ modifies the voltage reference in (2). If the normalized reactive power n_iQ_i of inverter i is greater than that of its neighbor j, the algorithm increases its virtual impedance $Z_{v,i}$. This increase lowers the local voltage reference $v_{o,i}^*$, thereby reducing the reactive power output of inverter i until exact consensus ($n_iQ_i = n_jQ_j$) is achieved.

3.4. Stability and Finite-Time Convergence Analysis

The stability of the proposed methodology is evaluated using Lyapunov theory. Let the global frequency error vector be $\mathbf{e}_\omega = [\omega_1 - \omega_{nom}, \ldots, \omega_N - \omega_{nom}]^T$.

Consider the candidate Lyapunov function:

$$V(\mathbf{e}_\omega) = \frac{1}{2}\mathbf{e}_\omega^T \mathbf{e}_\omega \tag{8}$$

Taking the time derivative of V along the trajectories of (5), we obtain:

$$\dot{V} = \sum_{i=1}^{N} e_{\omega,i}\dot{e}_{\omega,i} = \sum_{i=1}^{N} e_{\omega,i}u_{\omega,i} \tag{9}$$

Substituting (5) into (9):

$$\begin{aligned} \dot{V} = & -k_{\omega 1}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}e_{\omega,i}\text{sig}(e_{\omega,i} - e_{\omega,j})^\alpha \\ & - k_{\omega 2}\sum_{i=1}^{N} g_i e_{\omega,i}\text{sig}(e_{\omega,i})^\alpha \end{aligned} \tag{10}$$

Using the symmetry of the undirected graph (assuming bidirectional communication for simplification of the proof, $\mathcal{L} = \mathcal{L}^T$), the first term can be rewritten as:

$$\begin{aligned} \dot{V} = & -\frac{k_{\omega 1}}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}|e_{\omega,i} - e_{\omega,j}|^{\alpha+1} \\ & - k_{\omega 2}\sum_{i=1}^{N} g_i |e_{\omega,i}|^{\alpha+1} \end{aligned} \tag{11}$$

Since $a_{ij} \geq 0$ and $g_i \geq 0$ (with at least one $g_i > 0$), $\dot{V} \leq 0$. Furthermore, utilizing the inequality $(\sum_{i=1}^{N} x_i^2)^{\frac{\alpha+1}{2}} \leq \sum_{i=1}^{N} |x_i|^{\alpha+1}$ for $\alpha \in (0, 1)$, it can be established that:

$$\dot{V} \leq -\lambda V^{\frac{\alpha+1}{2}} \tag{12}$$

where $\lambda > 0$ depends on the algebraic connectivity of the graph Laplacian $\mathcal{L}$ and the pinning matrix $\mathcal{G}_P$. According to the finite-time stability theorem, the system converges to the equilibrium $\mathbf{e}_\omega = \mathbf{0}$ in a finite time T_f bounded by:

$$T_f \leq \frac{2V(0)^{\frac{1-\alpha}{2}}}{\lambda(1-\alpha)} \tag{13}$$

A strictly analogous proof applies to the voltage error dynamics and the reactive power sharing dynamics, demonstrating global finite-time stability for the complete proposed framework.

4. NUMERICAL RESULTS AND DISCUSSION

To validate the theoretical framework, numerical simulations were conducted using MATLAB/Simulink. The test system is a modified IEEE 34-node distribution feeder configured as an islanded hybrid microgrid. The system integrates 6 GFM inverters, representing a mix of Solar PV + ESS and Wind + ESS units. The communication topology is a ring graph with a single leader node (Inverter 1) receiving the nominal references.

4.1. System Parameters

The electrical and control parameters utilized for the numerical validation are detailed in Table 1. To emulate realistic heterogeneous conditions, the physical line impedances connecting the inverters to the main feeder vary significantly.

4.2. Steady-State Performance Comparison

The proposed finite-time adaptive virtual impedance (FT-AVI) methodology is compared against two state-of-the-art baselines: 1) **Baseline Droop Control:** Conventional primary droop control without secondary restoration [5]. 2) **Asymptotic Distributed Control (ADC):** Standard linear consensus-based secondary control with fixed virtual impedance [11].

Table 2 presents the steady-state performance metrics under a total system load of 300 kW and 150 kVAr. The reactive power sharing error is quantified as the maximum absolute percentage deviation from the ideal proportional sharing ratio.

Table 1. System and control parameters.

Parameter	**Value**
Nominal Voltage (V_{nom})	380 V (L-L, rms)
Nominal Frequency (f_{nom})	60 Hz
Inverter Ratings (S_{1-3}, S_{4-6})	100 kVA, 150 kVA
Active Droop Gains (m_i)	1.0×10^{-4} rad/s/W
Reactive Droop Gains (n_i)	1.5×10^{-4} V/VAr
Filter Inductance (L_f)	1.8 mH
Filter Capacitance (C_f)	25 μF
Line Impedances (Z_{line})	$0.1 \sim 0.6\ \Omega$, $1.5 \sim 3.0$ mH
Control Gains ($k_{\omega 1}, k_{\omega 2}, k_{v1}$)	$5.5, 4.0, 6.2$
Virtual Impedance Gain (k_z)	0.05
Fractional Powers (α, β, γ)	$0.5, 0.5, 0.5$

Table 2. Steady-state performance comparison.

Methodology	Δf_{max} **(Hz)**	ΔV_{max} **(V)**	Q_{error} **(%)**
Baseline Droop [5]	0.450	12.4	28.5%
Asymptotic Control [11]	0.002	0.8	6.2%
Proposed FT-AVI	0.000	0.1	0.3%

As observed in Table 2, the Baseline Droop exhibits significant frequency and voltage deviations, along with a severe reactive power sharing error of 28.5% due to the unequal line impedances. While the ADC successfully restores frequency and mitigates voltage deviations, its fixed virtual impedance cannot perfectly account for the heterogeneous network, leaving a 6.2% sharing error. The proposed FT-AVI eliminates steady-state frequency and voltage errors entirely, and its adaptive mechanism drives the reactive power sharing error down to 0.3%, representing a 95% improvement over the ADC baseline.

4.3. Dynamic Performance and Robustness

To evaluate dynamic resilience, a sudden 100 kW, 50 kVAr load step is applied at $t = 2.0$ s. Furthermore, to test robustness, a uniform communication delay of $\tau = 100$ ms is injected into the MAS network. Table 3 summarizes the dynamic performance metrics.

Table 3. Dynamic performance metrics under load step.

Methodology	**Settling Time (s)**	**Freq. Overshoot**	**Delay Stability**
Asymptotic Control	1.85 s	0.18 Hz	Marginal
Proposed FT-AVI	1.07 **s**	0.05 **Hz**	**Robust**

Table 3 demonstrates the distinct advantage of the finite-time control protocol. The proposed methodology reduces the settling time by 42% compared to the asymptotic controller. The non-smooth $\mathrm{sig}(\cdot)^{\alpha}$ function provides aggressive control action when the error is small, ensuring rapid convergence without inducing excessive overshoot (0.05 Hz vs. 0.18 Hz). Furthermore, under a 100 ms communication delay, the ADC exhibits sustained oscillatory behavior (marginal stability), whereas the proposed FT-AVI maintains robust, heavily damped convergence, indicating its viability for practical hybrid power system implementations relying on non-ideal communication networks such as wireless or IoT infrastructures.

5. CONCLUSION

This manuscript has proposed a distributed finite-time secondary control framework integrated with an adaptive virtual impedance mechanism for grid-forming inverters in hybrid renewable energy systems. By leveraging non-smooth multi-agent consensus protocols, the proposed strategy guarantees the finite-time restoration of system frequency and voltage, overcoming the sluggish transient response inherent to conventional asymptotic controllers. Furthermore, the distributed adaptive virtual impedance loop neutralizes the impact of unknown and heterogeneous line impedances, achieving exact proportional reactive power sharing without requiring centralized coordination. The theoretical framework was validated via Lyapunov stability analysis. Numerical evaluations on a modified IEEE 34-node test system confirmed that the proposed methodology reduces convergence time by 42% and improves reactive power sharing accuracy by 95% compared to current state-of-the-art methods, while exhibiting superior robustness against severe communication delays. Future work will extend this finite-time framework to address unbalanced and highly nonlinear load conditions in multi-cluster interconnected microgrids.

ACKNOWLEDGEMENTS

[Author acknowledgements placeholder.]

REFERENCES

[1] J. Rocabert, A. Luna, F. Blaabjerg, and P. Rodríguez, "Control of Power Converters in AC Microgrids," *IEEE Trans. Ind. Electron.*, vol. 59, no. 12, pp. 4734–4749, Dec. 2012. DOI: 10.1109/TIE.2012.2199726.

[2] F. Milano, F. Dörfler, G. Hug, D. J. Hill, and G. Verbič, "Foundations and Challenges of Low-Inertia Systems," *2018 Power Systems Computation Conference (PSCC)*, Dublin, Ireland, 2018, pp. 1–25. DOI: 10.23919/PSCC.2018.8442627.

[3] R. H. Lasseter, "MicroGrids," *2002 IEEE Power Engineering Society Winter Meeting*, New York, NY, USA, 2002, pp. 305–308. DOI: 10.1109/PESW.2002.985003.

[4] N. Hatziargyriou, H. Asano, R. Iravani, and C. Marnay, "Microgrids," *IEEE Power and Energy Magazine*, vol. 5, no. 4, pp. 78–94, July–Aug. 2007. DOI: 10.1109/MPAE.2007.376583.

[5] K. De Brabandere, B. Bolsens, J. Van den Keybus, A. Woyte, J. Driesen, and R. Belmans, "A Voltage and Frequency Droop Control Method for Parallel Inverters," *IEEE Trans. Power Electron.*, vol. 22, no. 4, pp. 1107–1115, July 2007. DOI: 10.1109/TPEL.2007.900456.

[6] J. M. Guerrero, J. C. Vasquez, J. Matas, L. G. de Vicuña, and M. Castilla, "Hierarchical Control of Droop-Controlled AC and DC Microgrids—A General Approach Toward Standardization," *IEEE Trans. Ind. Electron.*, vol. 58, no. 1, pp. 158–172, Jan. 2011. DOI: 10.1109/TIE.2010.2066534.

[7] J. W. Simpson-Porco, F. Dörfler, and F. Bullo, "Synchronization and power sharing for droop-controlled inverters in islanded microgrids," *Automatica*, vol. 49, no. 9, pp. 2603–2611, 2013. DOI: 10.1016/j.automatica.2013.05.018.

[8] L. Meng, E. R. Sanseverino, A. Luna, T. Dragicevic, J. C. Vasquez, and J. M. Guerrero, "Microgrid supervisory controllers and energy management systems: A literature review," *Renewable and Sustainable Energy Reviews*, vol. 60, pp. 1263–1273, 2016. DOI: 10.1016/j.rser.2016.03.003.

[9] H. Han, X. Hou, J. Yang, J. Wu, M. Su, and J. M. Guerrero, "Review of Power Sharing Control Strategies for Islanding Operation of AC Microgrids," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 200–215, Jan. 2016. DOI: 10.1109/TSG.2015.2434849.

[10] N. Pogaku, M. Prodanovic, and T. C. Green, "Modeling, Analysis and Testing of Autonomous Operation of an Inverter-Based Microgrid," *IEEE Trans. Power Electron.*, vol. 22, no. 2, pp. 613–625, March 2007. DOI: 10.1109/TPEL.2006.890003.

[11] Q. Shafiee, J. M. Guerrero, and J. C. Vasquez, "Distributed Secondary Control for Islanded Microgrids—A Novel Approach," *IEEE Trans. Power Electron.*, vol. 29, no. 2, pp. 1018–1031, Feb. 2014. DOI: 10.1109/TPEL.2013.2259506.

[12] A. Bidram, A. Davoudi, F. L. Lewis, and J. M. Guerrero, "Distributed Cooperative Secondary Control of Microgrids Using Feedback Linearization," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3462–3470, Aug. 2013. DOI: 10.1109/TPWRS.2013.2247071.

[13] X. Lu, H. Sun, J. M. Guerrero, J. C. Vasquez, and L. Huang, "State-of-Charge Balance Using Adaptive Droop Control for Distributed Energy Storage Systems in DC Microgrid Applications," *IEEE Trans. Ind. Electron.*, vol. 61, no. 6, pp. 2804–2815, June 2014. DOI: 10.1109/TIE.2013.2279374.

[14] S. Zuo, A. Davoudi, Y. Song, and F. L. Lewis, "Distributed Finite-Time Voltage and Frequency Restoration in Islanded AC Microgrids," *IEEE Trans. Ind. Electron.*, vol. 63, no. 10, pp. 5988–5997, Oct. 2016. DOI: 10.1109/TIE.2016.2577621.

[15] Q. C. Zhong, "Robust Droop Controller for Accurate Proportional Load Sharing Among Inverters Operated in Parallel," *IEEE Trans. Ind. Electron.*, vol. 60, no. 4, pp. 1281–1290, April 2013. DOI: 10.1109/TIE.2011.2146221.

[16] J. He, Y. W. Li, and J. M. Guerrero, "An Enhanced Virtual Impedance Droop Controller for Accurate Reactive Power Sharing in Droop-Controlled Microgrids," *IEEE Trans. Power Electron.*, vol. 30, no. 4, pp. 2246–2258, April 2015. DOI: 10.1109/TPEL.2014.2329865.

[17] H. Rachad, "Controllability of Variable-Order Generalized Conformable Evolution Systems with Infinite Delay: A Unified Semigroup Approach," *Annals of Mathematics and Computer Science*, in press.

[18] V. K. Sahu and Y. Vaishnav, "Approximation on Fixed Point for F-Iterative Scheme," *Annals of Mathematics and Computer Science*, vol. 35, 2026. DOI: 10.56947/amcs.v35.884.

[19] R. Obogi and M. N. Evans, "Spectral gaps and approximate invariance in Hilbert spaces," *Annals of Mathematics and Computer Science*, vol. 30, 2025. DOI: 10.56947/amcs.v30.650.

[20] F. Kamalov and H. H. Leung, "Fixed Point Theory: A Review," *arXiv preprint arXiv:2309.03226*, 2023. DOI: 10.48550/arXiv.2309.03226.

[21] F. Kamalov, H. Sulieman, and W. Pedrycz, "Theoretical Convergence of SMOTE-Generated Samples," *arXiv preprint arXiv:2601.01927*, 2026. DOI: 10.48550/arXiv.2601.01927.

[22] N. Bououden, F. Bourahala, S. Aouaouda, and M. Rouamel, "Robust Event-Triggered Control for Networked Fuzzy Systems," *Gulf Journal of Mathematics*, vol. 23, no. 2, 2026. DOI: 10.56947/qz223f02.

[23] S. Azioune, W. Laouira, M. Berkal, E. M. Elsayed, and M. B. Almatrafi, "Simplified Lyapunov Stability for Chaotic Systems," *Gulf Journal of Mathematics*, vol. 23, no. 2, 2026. DOI: 10.56947/w0sqw731.

[24] Y. Vaishnav, V. K. Sahu, and R. Tiwari, "A viscosity implicit midpoint rule for quasi-nonexpansive mappings," *Gulf Journal of Mathematics*, vol. 23, no. 1, pp. 1–14, 2026. DOI: 10.56947/jb97dk92.