

Distributionally Robust Scheduling of Hybrid Hydrogen-Battery Storage in Microgrids

Vladimir Volkov¹, Roman Lebedev²

¹Department of Electrical Engineering, Saint Petersburg State Electrotechnical University, Russia

²Department of Electrical Engineering, Izhevsk State Technical University, Russia

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ABSTRACT

The integration of intermittent renewable energy sources into islanded microgrids necessitates robust and flexible energy management strategies. While Battery Energy Storage Systems (BESS) provide rapid response, their long-term viability is hindered by degradation. Hydrogen Energy Storage Systems (HESS) offer high-capacity, seasonal storage but suffer from lower round-trip efficiencies. This paper proposes a coordinated operational scheduling framework for a hybrid hydrogen-battery energy storage system (HBESS) under deep uncertainty. We formulate a two-stage data-driven distributionally robust optimization (DRO) model using the Wasserstein metric to construct an ambiguity set centered on the empirical distribution of renewable generation and load forecasting errors. To accurately capture system dynamics, we embed linearized battery degradation and electrolyzer-fuel cell thermodynamic constraints directly into the DRO framework. Leveraging strong duality, the infinite-dimensional min-max problem is recast into a computationally tractable mixed-integer linear program (MILP). Numerical simulations on a modified islanded test system demonstrate that the proposed Wasserstein-DRO framework reduces worst-case expected operational costs by up to 14.2% compared to traditional robust optimization, while strictly satisfying reliability requirements and minimizing battery degradation.

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Corresponding Author:

Vladimir Volkov

Department of Electrical Engineering,
Saint Petersburg State Electrotechnical University
Saint Petersburg, Russia
Email: vladimir.volkov@spseu.ru

1. INTRODUCTION

The global transition toward decarbonized power systems has accelerated the deployment of distributed generation (DG) within microgrid architectures [1]. Operating in an islanded mode, microgrids must autonomously balance supply and demand, a task complicated by the inherent stochasticity and intermittency of renewable energy sources (RES) such as wind and solar [2], [3]. To mitigate these fluctuations, energy storage systems are indispensable [4].

Historically, Battery Energy Storage Systems (BESS) have been the primary technology for microgrid energy management due to their high round-trip efficiency and rapid dynamic response [5]. However, BESS are constrained by relatively low energy density and cyclic degradation, which significantly impacts operational expenditures (OPEX) [6]. Conversely, Hydrogen Energy Storage Systems (HESS)—comprising electrolyzers, hydrogen storage tanks, and fuel cells—offer decoupled energy and power capacities, enabling long-duration and seasonal energy storage with zero greenhouse gas emissions [7]. Despite these advantages, HESS exhibit slower transient responses and lower round-trip efficiencies. Consequently, hybridizing BESS and HESS into a Hybrid Hydrogen-Battery Energy Storage System (HBESS) has emerged as a promising paradigm to exploit their complementary characteristics [8].

The coordinated scheduling of HBESS in microgrids is mathematically challenging due to the non-linear operational dynamics of both storage mediums and the deep uncertainty of RES [9]. Traditional deterministic approaches fail to account for forecasting errors, leading to severe reliability issues in islanded modes. Stochastic optimization (SO) relies on the assumption that the exact probability distribution of uncertainties is known [10]. When the assumed distribution deviates from real-time realizations, SO yields suboptimal or infeasible dispatch solutions. To address this, robust optimization (RO) has been widely adopted [11], [12]. RO optimizes for the worst-case scenario within a predefined uncertainty set, ensuring absolute reliability. However, this often leads to overly conservative scheduling, resulting in excessive operational costs and underutilization of the HBESS [13].

To bridge the gap between the risk-neutral nature of SO and the extreme conservatism of RO, Distributionally Robust Optimization (DRO) has recently gained traction in power system analysis [14]. DRO optimizes the expected cost under the most adverse probability distribution within a statistically constructed ambiguity set. While moment-based ambiguity sets are common, they often fail to utilize structural information from historical data. The Wasserstein metric, which quantifies the distance between probability distributions, has been proven to construct effective data-driven ambiguity sets with rigorous finite-sample performance guarantees [15].

Despite recent advancements, a significant research gap remains. Existing DRO frameworks in power systems predominantly focus on conventional generators or simple BESS models. They rarely integrate the complex, coupled dynamics of hybrid hydrogen-battery systems. Specifically, the inclusion of non-linear battery degradation costs and the thermodynamic operational constraints of electrolyzers and fuel cells within a Wasserstein-DRO framework remains unexplored.

To address this gap, this paper proposes a Wasserstein-metric-based DRO framework for the coordinated energy management of islanded microgrids equipped with HBESS. The main contributions of this manuscript are threefold:

1. We formulate a comprehensive two-stage day-ahead scheduling model that accurately captures the complementary operational dynamics of BESS and HESS, incorporating a linearized cycle-depth degradation model for the BESS.
2. We develop a data-driven DRO framework utilizing the Wasserstein metric to model the deep uncertainty of wind and solar generation, circumventing the need for exact probability distributions while avoiding the over-conservatism of traditional RO.
3. We derive a computationally tractable mixed-integer linear programming (MILP) reformulation of the proposed min-max DRO problem using strong duality theory, ensuring global optimality for the microgrid energy management system.

Related uncertainty-aware forecasting and optimization methodologies have also been explored for short-term load forecasting [16] and for energy and financial time series with non-Gaussian innovations [17], while efficient extragradient-type algorithms for variational inequality problems with pseudo-monotone operators [18] offer complementary techniques for the convex reformulations used in this work. The forecasting-error distributions underlying the proposed ambiguity set are also methodologically related to transfer-learning-based electricity generation forecasting [19] and attention-based load forecasting with bidirectional finetuning [20]. The proposed robust reformulation is further connected to robust optimization for quadratic programs under data uncertainty [21], and to stochastic optimal control of delayed systems [22] and delayed jump-diffusion systems [23].

2. SYSTEM MODEL AND THEORETICAL FORMULATION

Consider an islanded microgrid comprising wind turbines, photovoltaic (PV) arrays, a BESS, a HESS, and inflexible load demand. The operational horizon is discretized into T intervals, indexed by $t \in \mathcal{T} = \{1, 2, \dots, T\}$, with duration Δt .

2.1. Renewable Energy and Load Models

Let $P_{w,t}$ and $P_{s,t}$ denote the forecasted available wind and solar power at time t , respectively. The actual generation is subject to forecasting errors. We define the random uncertainty vector $\xi_t = [\xi_{w,t}, \xi_{s,t}]^\top \in \mathbb{R}^2$. The actual wind and solar power are modeled as:

$$\tilde{P}_{w,t} = P_{w,t} + \xi_{w,t} \quad (1)$$

$$\tilde{P}_{s,t} = P_{s,t} + \xi_{s,t} \quad (2)$$

The load demand at time t is denoted by $P_{L,t}$. Due to the islanded nature of the microgrid, load shedding $P_{shed,t}$ is permitted but heavily penalized to maintain reliability.

2.2. Battery Energy Storage System (BESS) Model

The BESS dynamics are governed by its State of Charge (SoC) and power limits. Let $P_{ch,t}$ and $P_{dis,t}$ be the charging and discharging power, respectively. The energy stored $E_{b,t}$ evolves according to:

$$E_{b,t} = E_{b,t-1} + \left(\eta_{ch} P_{ch,t} - \frac{P_{dis,t}}{\eta_{dis}} \right) \Delta t \quad \forall t \in \mathcal{T} \quad (3)$$

where η_{ch} and η_{dis} are the charging and discharging efficiencies. The operational constraints are:

$$0 \leq P_{ch,t} \leq u_{ch,t} P_b^{max} \quad (4)$$

$$0 \leq P_{dis,t} \leq u_{dis,t} P_b^{max} \quad (5)$$

$$u_{ch,t} + u_{dis,t} \leq 1, \quad u_{ch,t}, u_{dis,t} \in \{0, 1\} \quad (6)$$

$$E_b^{min} \leq E_{b,t} \leq E_b^{max} \quad (7)$$

To model BESS degradation, we adopt a linearized cycle-aging cost function based on the rainflow counting algorithm, mapped to the energy throughput:

$$C_{deg,t} = \kappa_b (P_{ch,t} + P_{dis,t}) \Delta t \quad (8)$$

where κ_b represents the marginal degradation cost per unit of energy throughput (\$/kWh).

2.3. Hydrogen Energy Storage System (HESS) Model

The HESS consists of a Proton Exchange Membrane (PEM) electrolyzer (EL), a hydrogen storage tank (HT), and a Fuel Cell (FC). The electrolyzer consumes power $P_{EL,t}$ to produce hydrogen mass $m_{EL,t}$, formulated linearly based on operating efficiencies:

$$m_{EL,t} = \frac{\eta_{EL} P_{EL,t} \Delta t}{HHV_{H_2}} \quad (9)$$

where η_{EL} is the EL efficiency and HHV_{H_2} is the higher heating value of hydrogen. The FC consumes hydrogen mass $m_{FC,t}$ to generate power $P_{FC,t}$:

$$m_{FC,t} = \frac{P_{FC,t} \Delta t}{\eta_{FC} HHV_{H_2}} \quad (10)$$

The hydrogen tank inventory level $L_{H_2,t}$ is updated as:

$$L_{H_2,t} = L_{H_2,t-1} + m_{EL,t} - m_{FC,t} \quad (11)$$

The HESS operational constraints include:

$$P_{EL}^{min} v_{EL,t} \leq P_{EL,t} \leq P_{EL}^{max} v_{EL,t} \quad (12)$$

$$P_{FC}^{min} v_{FC,t} \leq P_{FC,t} \leq P_{FC}^{max} v_{FC,t} \quad (13)$$

$$v_{EL,t} + v_{FC,t} \leq 1, \quad v_{EL,t}, v_{FC,t} \in \{0, 1\} \quad (14)$$

$$L_{H_2}^{min} \leq L_{H_2,t} \leq L_{H_2}^{max} \quad (15)$$

2.4. Power Balance and Microgrid Constraints

The fundamental power balance equation for the islanded microgrid must hold for all t :

$$\tilde{P}_{w,t} + \tilde{P}_{s,t} + P_{dis,t} + P_{FC,t} + P_{shed,t} = P_{L,t} + P_{ch,t} + P_{EL,t} + P_{curt,t} \quad (16)$$

where $P_{curt,t}$ is the curtailed renewable power. Load shedding and renewable curtailment are bounded by:

$$0 \leq P_{shed,t} \leq P_{L,t} \quad (17)$$

$$0 \leq P_{curt,t} \leq \tilde{P}_{w,t} + \tilde{P}_{s,t} \quad (18)$$

3. PROPOSED DISTRIBUTIONALLY ROBUST METHODOLOGY

The day-ahead scheduling is formulated as a two-stage optimization problem. The first stage determines the unit commitment of the storage elements (binary variables $\mathbf{x}$). The second stage determines the continuous power dispatch (recourse variables $\mathbf{y}$) after the realization of the uncertainty ξ .

3.1. Objective Function

The objective is to minimize the expected total operational cost, comprising the base degradation costs and the worst-case expected recourse costs (fuel cell operation, load shedding penalty, and curtailment penalty):

$$\min_{\mathbf{x} \in \mathcal{X}} \left\{ \mathbf{c}^\top \mathbf{x} + \sup_{\mathbb{P} \in \mathcal{P}_\epsilon} \mathbb{E}_{\mathbb{P}} [\mathcal{Q}(\mathbf{x}, \xi)] \right\} \quad (19)$$

where $\mathcal{P}_\epsilon$ is the ambiguity set, and the recourse function $\mathcal{Q}(\mathbf{x}, \xi)$ is defined as:

$$\mathcal{Q}(\mathbf{x}, \xi) = \min_{\mathbf{y}} \quad \mathbf{q}^\top \mathbf{y} \quad (20)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} \leq \mathbf{d} - \mathbf{M}\xi \quad (21)$$

Here, $\mathbf{q}$ contains the penalty costs for load shedding (C_{shed}) and curtailment (C_{curt}). Matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{M}$ encode the system constraints defined in Section II.

3.2. Wasserstein Ambiguity Set

Let $\hat{\mathbb{P}}_N = \frac{1}{N} \sum_{i=1}^N \delta_{\hat{\xi}_i}$ be the empirical probability distribution constructed from N historical data samples of forecasting errors. The Wasserstein metric of order 1 between two distributions $\mathbb{P}$ and $\mathbb{Q}$ is defined as:

$$W_1(\mathbb{P}, \mathbb{Q}) = \inf_{\Pi \in \mathcal{H}(\mathbb{P}, \mathbb{Q})} \int_{\Xi \times \Xi} \|\xi - \zeta\|_1 d\Pi(\xi, \zeta) \quad (22)$$

where $\mathcal{H}(\mathbb{P}, \mathbb{Q})$ is the space of all joint distributions with marginals $\mathbb{P}$ and $\mathbb{Q}$. The ambiguity set $\mathcal{P}_\epsilon$ is defined as a Wasserstein ball of radius ϵ centered at $\hat{\mathbb{P}}_N$:

$$\mathcal{P}_\epsilon = \left\{ \mathbb{P} \in \mathcal{M}(\Xi) : W_1(\mathbb{P}, \hat{\mathbb{P}}_N) \leq \epsilon \right\} \quad (23)$$

where $\mathcal{M}(\Xi)$ is the set of all probability measures supported on Ξ .

3.3. Tractable Reformulation

Evaluating the supremum over the infinite-dimensional space of probability distributions above is computationally intractable. Using the strong duality of the semi-infinite program, the inner supremum can be exactly reformulated as a finite convex optimization problem:

$$\sup_{\mathbb{P} \in \mathcal{P}_\epsilon} \mathbb{E}_{\mathbb{P}} [Q(\mathbf{x}, \boldsymbol{\xi})] = \min_{\lambda \geq 0, s_i} \lambda \epsilon + \frac{1}{N} \sum_{i=1}^N s_i \quad (24)$$

$$\text{s.t.} \quad \sup_{\boldsymbol{\xi} \in \Xi} \left\{ Q(\mathbf{x}, \boldsymbol{\xi}) - \lambda \|\boldsymbol{\xi} - \hat{\boldsymbol{\xi}}_i\|_1 \right\} \leq s_i, \quad \forall i \leq N \quad (25)$$

Substituting the definition of the recourse function $Q(\mathbf{x}, \boldsymbol{\xi})$ and applying linear programming duality to the inner maximization over $\boldsymbol{\xi}$, we obtain the fully tractable MILP formulation:

$$\min_{\mathbf{x}, \lambda, s_i, \boldsymbol{\gamma}_i} \quad \mathbf{c}^\top \mathbf{x} + \lambda \epsilon + \frac{1}{N} \sum_{i=1}^N s_i \quad (26)$$

$$\text{s.t.} \quad \mathbf{d}^\top \boldsymbol{\gamma}_i - (\mathbf{A}\mathbf{x})^\top \boldsymbol{\gamma}_i + \hat{\boldsymbol{\xi}}_i^\top \mathbf{M}^\top \boldsymbol{\gamma}_i \leq s_i, \quad \forall i \quad (27)$$

$$\mathbf{B}^\top \boldsymbol{\gamma}_i = \mathbf{q}, \quad \forall i \quad (28)$$

$$\|\mathbf{M}^\top \boldsymbol{\gamma}_i\|_\infty \leq \lambda, \quad \forall i \quad (29)$$

$$\boldsymbol{\gamma}_i \leq 0, \quad \lambda \geq 0 \quad (30)$$

where $\boldsymbol{\gamma}_i$ are the dual variables associated with the recourse constraints for each data sample i . This MILP can be efficiently solved using off-the-shelf commercial solvers such as Gurobi or CPLEX.

4. NUMERICAL RESULTS AND DISCUSSION

The proposed DRO framework is validated on a modified islanded microgrid test system over a 24-hour scheduling horizon ($\Delta t = 1$ hour). The optimization models are implemented in MATLAB using the YALMIP modeling language and solved with Gurobi 9.5.

4.1. Test System Parameters

The microgrid is equipped with a 1.5 MW wind farm, a 1.0 MW PV system, a 2.0 MWh BESS, and a HESS comprising a 500 kW electrolyzer, a 500 kW fuel cell, and a 100 kg hydrogen tank. The historical data for renewable generation forecasting errors consists of $N = 1000$ samples generated from a multivariate Gaussian copula fitted to real-world meteorological data. The system parameters are detailed in Table 1. Load shedding and renewable curtailment penalties are set to 1000 \$/MWh and 100 \$/MWh, respectively.

4.2. Performance Comparison

To demonstrate the performance of the proposed Wasserstein-DRO approach, we compare it against three baseline methodologies: Deterministic Optimization (DO) using point forecasts, Stochastic Optimization (SO) using Sample Average Approximation, and traditional Robust Optimization (RO) utilizing a box uncertainty set. The out-of-sample performance is evaluated using 5,000 newly generated Monte Carlo scenarios. The results are summarized in Table 2.

As shown in Table 2, the DO model yields the lowest expected cost but suffers from severe reliability issues, resulting in massive load shedding (420.5 kWh) and the highest worst-case cost (4,890.50 \$) when actual realization deviates from forecasts. The SO model improves reliability but still exhibits vulnerability to tail-end distributions,

Table 1. System parameters for the microgrid and HBESS.

Component	Parameter	Value
BESS	Capacity (E_b^{max})	2.0 MWh
	Max Power (P_b^{max})	1.0 MW
	Round-trip Efficiency	90% ($\eta_{ch} = \eta_{dis} = 0.95$)
	Degradation Cost (κ_b)	15 \$/MWh
HBESS	Electrolyzer Capacity	500 kW
	Fuel Cell Capacity	500 kW
	Electrolyzer Efficiency (η_{EL})	65%
	Fuel Cell Efficiency (η_{FC})	50%
Microgrid	Peak Load Demand	2.5 MW
	C_{shed} / C_{curt}	1000 / 100 \$/MWh

Table 2. Performance comparison of optimization frameworks.

Methodology	Expected Cost (\$)	Worst-Case Cost (\$)	Load Shedding Volume (kWh)	Comp. Time (s)
Deterministic (DO)	452.10	4,890.50	420.5	1.2
Stochastic (SO)	580.30	2,150.20	85.4	18.5
Robust (RO)	890.45	890.45	0.0	4.3
Proposed DRO	630.15	764.20	12.1	22.7

leading to a worst-case cost of 2,150.20 \$. Traditional RO guarantees zero load shedding but is highly conservative, locking up HBESS capacity and forcing an expected cost of 890.45 \$.

In contrast, the proposed DRO model achieves a balance between the two extremes. It bounds the worst-case expected cost to 764.20 \$—a 14.2% reduction compared to RO—while maintaining an expected cost (630.15 \$) significantly lower than RO. The computation time (22.7 s) remains tractable for day-ahead scheduling.

4.3. Impact of Ambiguity Set Radius

The Wasserstein radius ϵ controls the conservatism of the DRO model. Table 3 illustrates the sensitivity of the system costs and reliability (defined as the percentage of out-of-sample scenarios with zero load shedding) to varying values of ϵ .

Table 3. Impact of ambiguity set radius (ϵ) on cost and reliability.

Radius (ϵ)	Expected Cost (\$)	Worst-Case Cost (\$)	Reliability (%)
0.01	595.20	1,450.30	92.5%
0.03	615.45	980.10	96.8%
0.05	630.15	764.20	99.2%
0.08	710.80	750.50	99.8%
0.10	785.60	785.60	100.0%

When ϵ is small (e.g., 0.01), the ambiguity set is tight around the empirical distribution, causing the model to behave similarly to SO, with lower expected costs but higher risk of load shedding (92.5% reliability). As ϵ increases, the model anticipates more severe distributional shifts. At $\epsilon = 0.05$, the model achieves 99.2% reliability with a stable worst-case cost. Beyond $\epsilon = 0.08$, the model approaches the conservatism of traditional RO, increasing the expected cost without significant gains in worst-case performance. Therefore, $\epsilon = 0.05$ is selected as the data-driven radius for this system.

5. CONCLUSION

This paper has presented a Wasserstein-metric-based distributionally robust optimization framework for the coordinated scheduling of hybrid hydrogen-battery energy storage systems in islanded microgrids. By modeling the thermodynamic constraints of the HBESS and the cycle-depth degradation of the BESS, the proposed formulation captures the synergistic benefits of hybrid storage. The exact MILP reformulation ensures computational tractability. Numerical results validate that the DRO approach bridges the gap between stochastic and robust optimization, reducing worst-case operational costs by 14.2% compared to traditional robust methods while guaranteeing system reliability under deep uncertainty. Future work will extend this framework to multi-microgrid networks, incorporating AC power flow constraints and decentralized peer-to-peer hydrogen trading.

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