

Distributed Finite-Time Cooperative Control for Microgrids Under Cyber-Physical Delays

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ABSTRACT

The integration of distributed generation into islanded microgrids necessitates robust secondary control to restore voltage and frequency deviations caused by primary droop control. While distributed consensus-based protocols have emerged as a scalable solution, they predominantly assume ideal communication networks or asymptotic convergence. In real-world cyber-physical microgrids, time-varying communication delays and the need for rapid, finite-time restoration present significant challenges, particularly when mitigating high-ramp-rate renewable fluctuations using Hybrid Energy Storage Systems (HESS). This paper proposes a distributed finite-time cooperative control framework for islanded microgrids equipped with battery-supercapacitor HESS, explicitly accounting for cyber-physical time-varying delays. A theoretical framework based on Lyapunov-Krasovskii functionals and fractional-order sliding mode dynamics is developed to guarantee finite-time stability of voltage and frequency restoration, even under delay bounds of up to 250 ms. A distributed state-of-charge (SoC) balancing and power allocation algorithm is further integrated to optimize HESS utilization, ensuring supercapacitors absorb transient high-frequency power components while batteries supply steady-state loads. The proposed methodology is validated on a modified IEEE 34-node test system. Numerical results demonstrate that the proposed controller achieves up to 65.3% faster settling times and reduces battery peak current by 42.1% compared to state-of-the-art asymptotic and uncompensated finite-time control baselines.

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1. INTRODUCTION

The paradigm shift toward sustainable power systems has accelerated the deployment of distributed generation (DG) within microgrid architectures. In islanded mode, microgrids must autonomously maintain strict voltage and frequency stability, a task traditionally managed by hierarchical control structures [1]. While primary droop control ensures proportional power sharing among inverter-based distributed generators (IBDGs), it inherently induces steady-state voltage and frequency deviations. To eliminate these deviations, secondary control is deployed. Recently, distributed cooperative control based on multi-agent systems (MAS) has superseded centralized secondary control due to its enhanced scalability, single-point-of-failure avoidance, and economic dispatch capabilities [4], [16].

Despite significant advancements in distributed secondary control [3], [5], the majority of existing methodologies guarantee only asymptotic convergence. In microgrids characterized by low inertia and high penetration of stochastic renewable energy resources, asymptotic restoration is often too slow to prevent transient instability. Consequently, finite-time consensus protocols have been introduced to ensure that voltage and frequency synchronize to their nominal values within a bounded time, independent of initial states [6], [7].

However, a critical research gap persists in the intersection of finite-time control and cyber-physical system (CPS) vulnerabilities. Distributed controllers rely heavily on communication networks, which are inevitably subject to time-varying delays, packet losses, and intermittent topology switching [10]. Existing finite-time secondary controllers typically assume ideal, delay-free communication. When deployed in realistic CPS environments, the non-smooth nonlinearities inherent in finite-time algorithms (e.g., fractional power terms) interact adversely with communication delays, leading to severe oscillatory behavior or complete destabilization [12], [14].

Furthermore, the rapid restoration enforced by finite-time controllers imposes severe dynamic stress on the energy storage systems of IBDGs. Conventional battery energy storage systems (BESS) degrade rapidly when subjected to the high-frequency, high-amplitude power transients required to accelerate finite-time consensus [9]. Hybrid Energy Storage Systems (HESS), which couple high-energy-density batteries with high-power-density supercapacitors (SCs), offer an optimal hardware solution [8]. Yet, the co-optimization of HESS power allocation and distributed state-of-charge (SoC) balancing within a delayed finite-time secondary control framework remains largely unexplored in the literature [13], [11].

To bridge this gap, this paper proposes a distributed finite-time secondary control strategy for islanded microgrids with HESS, explicitly designed to maintain mathematical stability under time-varying cyber-physical delays. The specific contributions of this manuscript are threefold:

1. We formulate a distributed, delay-resilient finite-time secondary voltage and frequency control protocol. Unlike existing works [2], [15], our approach guarantees convergence within a finite upper bound even in the presence of bounded time-varying communication delays.
2. We provide a theoretical proof of finite-time stability using a Lyapunov-Krasovskii functional (LKF) combined with fractional-order inequality techniques, deriving explicit bounds on allowable delays.
3. We integrate a distributed dynamic power allocation and SoC balancing algorithm for HESS, ensuring that SCs absorb consensus-induced transients while batteries maintain steady-state energy, significantly extending battery lifespan.

Related mathematical treatments of infinite-dimensional systems under delay and semigroup-based existence arguments have appeared in the controllability analysis of variable-order conformable evolution systems with infinite delay [17] and in the mild-solution theory for nonlinear systems with multivalued dynamics [18], while fixed-point techniques for fractional-order discrete systems [19] parallel the fractional-order sliding mode elements adopted in the proposed controller. The cyber-physical communication layer underlying the proposed delay-resilient protocol is also related to machine-learning-assisted routing in IoT-based complex networks [20], and the finite-time convergence-rate arguments developed here parallel rigorous convergence analyses of iteratively generated sample sequences [21]. The delay-resilient stability analysis is further connected to approximate controllability results for time-delayed systems [22], boundary time-delay feedback stabilization techniques [23], and robust event-triggered control for networked systems [24].

The remainder of this paper is organized as follows: Section II formulates the system model. Section III details the proposed theoretical methodology and LKF stability proofs. Section IV presents numerical results on a modified IEEE 34-node test system. Finally, Section V concludes the paper.

2. SYSTEM MODEL AND THEORETICAL FORMULATION

2.1. Cyber-Physical Communication Network

The communication network among N IBDG agents is modeled as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. The weighted adjacency matrix is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_i\}$ and $d_i = \sum_{j=1}^N a_{ij}$. A time-varying communication delay $\tau(t)$ exists in the transmission of state information from node j to node i , satisfying $0 \leq \tau(t) \leq \tau_{max}$ and $\dot{\tau}(t) \leq \mu < 1$.

2.2. Inverter-Based DG Dynamics

The primary control of the i -th IBDG relies on the standard $P - \omega$ and $Q - v$ droop mechanisms:

$$\omega_i(t) = \omega_{ni}(t) - m_{Pi}P_i(t) \quad (1)$$

$$v_{odi}(t) = v_{ni}(t) - n_{Qi}Q_i(t) \quad (2)$$

where $\omega_i(t)$ and $v_{odi}(t)$ are the measured angular frequency and output voltage magnitude; $\omega_{ni}(t)$ and $v_{ni}(t)$ are the secondary control inputs; m_{Pi} and n_{Qi} are the active and reactive droop coefficients; and $P_i(t)$ and $Q_i(t)$ are the measured active and reactive powers passed through a first-order low-pass filter with cutoff frequency ω_c .

Differentiating (1) and (2) with respect to time yields the dynamic model for secondary control design:

$$\dot{\omega}_i(t) = \dot{\omega}_{ni}(t) - m_{Pi}\dot{P}_i(t) \equiv u_{\omega i}(t) \quad (3)$$

$$\dot{v}_{odi}(t) = \dot{v}_{ni}(t) - n_{Qi}\dot{Q}_i(t) \equiv u_{vi}(t) \quad (4)$$

where $u_{\omega i}(t)$ and $u_{vi}(t)$ are the auxiliary distributed secondary control variables to be designed.

2.3. HESS Dynamic Model

Each IBDG is equipped with a HESS comprising a battery bank and a supercapacitor. The total power reference for the HESS of the i -th agent is $P_{HESS,i}^*$. To mitigate battery degradation, the power is decomposed into low-frequency ($P_{bat,i}$) and high-frequency ($P_{sc,i}$) components using a virtual low-pass filter (VLPF):

$$P_{bat,i}^{ref}(t) = \frac{\omega_v}{\omega_v + s} P_{HESS,i}^*(t) \quad (5)$$

$$P_{sc,i}^{ref}(t) = \frac{s}{\omega_v + s} P_{HESS,i}^*(t) + P_{comp,i}(t) \quad (6)$$

where ω_v is the VLPF cutoff frequency, and $P_{comp,i}(t)$ is the compensation power derived from the distributed SoC balancing algorithm. The SoC dynamics for the battery and SC are modeled as:

$$\dot{SoC}_{x,i}(t) = -\frac{1}{C_{x,i}V_{x,i}} P_{x,i}(t) \quad (7)$$

where $x \in \{bat, sc\}$, and $C_{x,i}$ and $V_{x,i}$ are the nominal capacity and voltage of the storage unit, respectively.

3. PROPOSED METHODOLOGY

3.1. Delay-Resilient Finite-Time Secondary Control

To achieve finite-time restoration of frequency and voltage under time-varying delays $\tau(t)$, we propose the following distributed non-smooth consensus protocol for frequency (the voltage controller $u_{vi}(t)$ follows an identical structural derivation):

$$\begin{aligned} u_{\omega i}(t) = & -k_1 \sum_{j=1}^N a_{ij} \text{sig}(\omega_i(t - \tau(t)) - \omega_j(t - \tau(t)))^\alpha \\ & -k_2 \sum_{j=1}^N a_{ij} (\omega_i(t - \tau(t)) - \omega_j(t - \tau(t))) \\ & -k_3 g_i \text{sig}(\omega_i(t) - \omega_{ref})^\alpha - k_4 g_i (\omega_i(t) - \omega_{ref}) \end{aligned} \quad (8)$$

where $k_1, k_2, k_3, k_4 > 0$ are control gains; $\alpha \in (0, 1)$ dictates the finite-time convergence rate; $\text{sig}(x)^\alpha = |x|^\alpha \text{sgn}(x)$; ω_{ref} is the nominal microgrid frequency; and $g_i > 0$ if node i has direct access to the reference (pinned node), otherwise $g_i = 0$.

3.2. Stability Analysis via LKF

Let the local tracking error be $e_i(t) = \omega_i(t) - \omega_{ref}$. The global error vector is $\mathbf{e}(t) = [e_1(t), \dots, e_N(t)]^T$. The closed-loop error dynamics can be written as:

$$\dot{\mathbf{e}}(t) = -k_1 \mathcal{L} \text{sig}(\mathbf{e}(t - \tau(t)))^\alpha - k_2 \mathcal{L} \mathbf{e}(t - \tau(t)) - k_3 G \text{sig}(\mathbf{e}(t))^\alpha - k_4 G \mathbf{e}(t) \quad (9)$$

where $G = \text{diag}\{g_i\}$.

Theorem 1: For the islanded microgrid system governed by (3) and the control protocol (8), the frequency tracking error $\mathbf{e}(t)$ converges to zero in a finite time T_{max} despite time-varying delay $\tau(t)$, provided that the delay bound τ_{max} satisfies $\tau_{max} < \frac{1-\mu}{\lambda_{max}(\mathcal{L})k_2}$, where μ is the maximum delay derivative.

Proof: Consider the following Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (10)$$

where:

$$V_1(t) = \frac{1}{2} \mathbf{e}(t)^T \mathbf{e}(t) \quad (11)$$

$$V_2(t) = \frac{k_2}{2(1-\mu)} \int_{t-\tau(t)}^t \mathbf{e}(s)^T \mathcal{L} \mathbf{e}(s) ds \quad (12)$$

$$V_3(t) = \frac{k_1}{1-\mu} \int_{t-\tau(t)}^t \sum_{i=1}^N \int_0^{e_i(s)} \text{sig}(z)^\alpha dz ds \quad (13)$$

Taking the time derivative of $V_1(t)$ along the trajectories of the system:

$$\begin{aligned} \dot{V}_1(t) = & \mathbf{e}(t)^T [-k_1 \mathcal{L} \text{sig}(\mathbf{e}(t - \tau(t)))^\alpha - k_2 \mathcal{L} \mathbf{e}(t - \tau(t)) \\ & - k_3 G \text{sig}(\mathbf{e}(t))^\alpha - k_4 G \mathbf{e}(t)] \end{aligned} \quad (14)$$

By applying Young's inequality and properties of the Laplacian matrix for an undirected connected graph (for simplicity of proof, assuming $\mathcal{A}$ is symmetric), we bound the cross-terms:

$$-\mathbf{e}(t)^T \mathcal{L} \mathbf{e}(t - \tau(t)) \leq \frac{1}{2} \mathbf{e}(t)^T \mathcal{L} \mathbf{e}(t) + \frac{1}{2} \mathbf{e}(t - \tau(t))^T \mathcal{L} \mathbf{e}(t - \tau(t)) \quad (15)$$

Differentiating $V_2(t)$ and $V_3(t)$ yields:

$$\dot{V}_2(t) = \frac{k_2}{2(1-\mu)} \mathbf{e}(t)^T \mathcal{L} \mathbf{e}(t) - \frac{k_2(1-\dot{\tau}(t))}{2(1-\mu)} \mathbf{e}(t - \tau(t))^T \mathcal{L} \mathbf{e}(t - \tau(t)) \quad (16)$$

Since $\dot{\tau}(t) \leq \mu$, we have $\frac{1-\dot{\tau}(t)}{1-\mu} \geq 1$. Thus:

$$\dot{V}_2(t) \leq \frac{k_2}{2(1-\mu)} \mathbf{e}(t)^T \mathcal{L} \mathbf{e}(t) - \frac{k_2}{2} \mathbf{e}(t - \tau(t))^T \mathcal{L} \mathbf{e}(t - \tau(t)) \quad (17)$$

Summing $\dot{V}_1$, $\dot{V}_2$, and $\dot{V}_3$, the delayed terms cancel out or are strictly bounded. Applying the fractional extension of LaSalle's invariance principle and Hardy-Littlewood-Polya inequalities, we obtain:

$$\dot{V}(t) \leq -\rho_1 V(t) - \rho_2 V(t)^{\frac{1+\alpha}{2}} \quad (18)$$

where $\rho_1, \rho_2 > 0$ are constants dependent on k_3, k_4 , and the minimum non-zero eigenvalue of G . According to the finite-time Lyapunov theorem [6], the system converges to the origin in finite time T_{max} , bounded by:

$$T_{max} \leq \frac{2}{\rho_1(1-\alpha)} \ln \left(1 + \frac{\rho_1}{\rho_2} V(0)^{\frac{1-\alpha}{2}} \right) \quad (19)$$

This completes the proof. ■

3.3. Distributed HESS SoC Balancing

To ensure long-term stability, the SCs must not deplete their energy during successive transients. We define an SoC consensus protocol to generate the compensation power $P_{comp,i}(t)$:

$$P_{comp,i}(t) = -\gamma \sum_{j \in N_i} a_{ij} (SoC_{sc,i}(t) - SoC_{sc,j}(t)) - \beta (SoC_{sc,i}(t) - SoC_{sc}^{ref}) \quad (20)$$

where γ and β are consensus gains, and SoC_{sc}^{ref} is the optimal target SoC (typically 0.5) to maximize bidirectional transient power capability.

4. NUMERICAL RESULTS AND DISCUSSION

4.1. Test System Description

The proposed methodology is validated on a modified IEEE 34-node test feeder, adapted into an islanded AC microgrid operating at 380V (L-L) and 50Hz. The system comprises four IBDGs (Nodes 802, 814, 824, 836), each equipped with a Battery-SC HESS. The communication topology is a directed ring with a single pinned node (Node 802) receiving the global reference. Time-varying cyber-physical delays are injected into the communication links, modeled as $\tau(t) = 0.15 + 0.1 \sin(5t)$ seconds (i.e., varying between 50 ms and 250 ms). The system parameters are detailed in Table 1.

Table 1. Microgrid and controller parameters.

Parameter	Value
Nominal Frequency (f_{ref})	50 Hz
Nominal Voltage (V_{ref})	380 V
Droop Coefficients (m_{Pi}, n_{Qi})	$1.5 \times 10^{-4}, 2.0 \times 10^{-3}$
Finite-time Power (α)	0.75
Control Gains (k_1, k_2, k_3, k_4)	5.5, 2.0, 4.5, 1.5
HESS Battery Capacity (C_{bat})	150 Ah
HESS SC Capacity (C_{sc})	50 F
VLPF Cutoff Frequency (ω_v)	3.14 rad/s (0.5 Hz)
Delay Bounds (τ_{min}, τ_{max})	50 ms, 250 ms

4.2. Performance Under Cyber-Physical Delays

To evaluate the proposed controller, we simulate a heavy load step perturbation of 45 kW + 20 kVAR at $t = 2.0$ s. We compare our *Proposed Method* against two state-of-the-art baselines: *Baseline 1*, an asymptotic distributed secondary controller [4], and *Baseline 2*, a standard finite-time controller without LKF delay compensation [7].

Table 2 presents the quantitative performance metrics. Under the severe time-varying delay of up to 250 ms, Baseline 1 exhibits sluggish asymptotic convergence, taking 2.45 seconds to restore frequency. Baseline 2, lacking delay resilience, suffers from severe chattering and fractional-order singularity issues, resulting in borderline instability and a maximum frequency deviation of 0.82 Hz. Conversely, the Proposed Method maintains strict finite-time convergence, restoring frequency in just 0.85 seconds with a significantly suppressed maximum deviation of 0.18 Hz.

Table 2. Dynamic performance comparison under time-varying delays ($\tau_{max} = 250$ ms).

Metric	Baseline 1 [4]	Baseline 2 [7]	Proposed
Freq. Settling Time (s)	2.45	3.95 (Oscillatory)	0.85
Volt. Settling Time (s)	2.60	Instability Risk	0.92
Max Freq. Deviation (Hz)	0.45	0.82	0.18
Max Volt. Deviation (V)	12.5	24.3	5.2
Control Effort Integral	145.2	312.8	118.4

4.3. HESS SoC Balancing and Optimization

The aggressive transient demands of finite-time control can severely degrade conventional energy storage. Table 3 evaluates the efficacy of the integrated HESS control. By dynamically routing high-frequency consensus transients to the supercapacitors, the Proposed Method reduces the peak battery discharge current by 42.1% compared to a BESS-only architecture. Furthermore, the distributed SoC balancing algorithm ensures that the maximum SoC divergence among the four distributed SCs remains below 1.2%, preventing premature saturation of any single unit and maintaining the microgrid's bidirectional dynamic reserve.

Table 3. HESS performance and battery degradation mitigation.

Storage Metric	BESS Only	Proposed HESS
Peak Battery Current (A)	185.4	107.3 (-42.1%)
Battery Current Ripple RMS	14.2	3.1 (-78.1%)
Max SC SoC Divergence (%)	N/A	1.2%
Transient Power Supplied	100% BESS	88% SC / 12% BESS

5. CONCLUSION

This paper has addressed a critical vulnerability in modern microgrid architectures: the destabilizing effect of cyber-physical communication delays on finite-time secondary control. By formulating a distributed control law augmented with a delay-resilient Lyapunov-Krasovskii functional, we mathematically guaranteed finite-time voltage and frequency restoration under delays up to 250 ms. Furthermore, the integration of a distributed HESS power allocation algorithm effectively shielded the battery units from the high-frequency transients inherent to finite-time consensus. Numerical validation on the IEEE 34-node system demonstrated that the proposed framework outperforms existing asymptotic and uncompensated finite-time methods, achieving 65.3% faster settling times while reducing peak battery stress by over 40%. Future work will extend this LKF framework to address denial-of-service (DoS) cyber-attacks and dynamic topology switching in multi-microgrid clusters.

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