Journal of Modern Energy Research
Vol. 3, Mar 2025, pp. 19–24
ISSN: 3078-3534, DOI: 10.56947/jmer.v3.4

Adaptive MPC-Based Grid-Forming Control for Hybrid Renewable Storage Systems

Li Jun[1], Wang Yong[2]

[1]Department of Electrical Engineering, Shanghai Dianji University, China
[2]Department of Electrical Engineering, Shaoguan University, China

Article Info

Article history:
Received Jan 21, 2025
Revised Mar 4, 2025
Accepted Mar 12, 2025

Keywords:
Virtual synchronous generator (VSG); Battery state-of-charge management; Rate of change of frequency (RoCoF); Linear parameter-varying modeling; Quadratic programming; Low-inertia power grids

ABSTRACT

The paradigm shift toward 100% renewable energy systems has precipitated a critical decline in rotational inertia, rendering modern power grids highly susceptible to frequency instability and high Rates of Change of Frequency (RoCoF). Grid-forming (GFM) inverters utilizing Virtual Synchronous Generator (VSG) control offer a promising solution; however, existing methodologies predominantly employ fixed virtual inertia and damping coefficients, which fail to exploit the dynamic capabilities of hybrid renewable energy systems (HRES). This paper addresses this gap by proposing a Linear Parameter-Varying Model Predictive Control (LPV-MPC) framework for adaptive VSG control in hybrid solar photovoltaic (PV), wind, and battery energy storage systems (BESS). The proposed methodology dynamically optimizes virtual inertia and damping in real time by formulating the control objective as a constrained Quadratic Program (QP). The optimization explicitly accounts for the stochastic generation limits of the PV-wind system, inverter capacity constraints, and the real-time State-of-Charge (SoC) of the BESS. By embedding the nonlinear VSG dynamics into an LPV state-space model, the proposed MPC guarantees optimal transient performance while preventing BESS degradation. Numerical validation on a modified IEEE 39-bus test system demonstrates that the proposed LPV-MPC adaptive VSG mitigates maximum frequency deviation by 38% and RoCoF by 42% compared to conventional fixed-parameter VSG, ensuring robust operation under severe low-inertia conditions and asymmetrical faults.

Corresponding Author:
Li Jun
Department of Electrical Engineering,
Shanghai Dianji University
Shanghai, China
Email: li.jun@sdju.edu.cn

1. INTRODUCTION

The accelerated integration of inverter-based resources (IBRs), such as solar photovoltaic (PV) and wind energy conversion systems, is fundamentally transforming the dynamic characteristics of modern power systems [1], [2]. Unlike conventional synchronous generators (SGs) that intrinsically couple their kinetic energy to the grid via synchronous rotation, IBRs are decoupled from the grid by power electronic converters. Consequently, the displacement of SGs by IBRs leads to a drastic reduction in total system inertia [3], [4]. In such ultra-low inertia grids, minor power imbalances can provoke severe frequency excursions and unacceptably high Rates of Change of Frequency (RoCoF), potentially triggering cascaded outages and system blackouts [5].

To mitigate these stability challenges, grid-forming (GFM) inverters—particularly those implementing Virtual Synchronous Generator (VSG) or synchronverter topologies—have emerged as a pivotal technology [6], [7]. VSG control algorithms emulate the swing equation of traditional SGs, thereby providing synthetic inertia and damping to the grid [8]. However, a fundamental limitation of the conventional VSG paradigm is the reliance on fixed virtual inertia (J) and damping (D) coefficients [9], [10]. While fixed parameters can be tuned for a specific operating point, they are highly suboptimal during severe transients. Furthermore, existing VSG implementations often assume an infinite DC-link energy source, ignoring the stringent physical and operational constraints of the underlying hybrid renewable energy system (HRES) and Battery Energy Storage System (BESS) [11], [12].

Recent literature has explored adaptive VSG control to dynamically adjust J and D based on local frequency measurements [13], [14]. For instance, bang-bang control and fuzzy logic-based adaptations have been proposed to increase inertia during acceleration and decrease it during deceleration. Despite their merits, these heuristic approaches lack mathematical optimality and fail to systematically incorporate multi-variable constraints, such as inverter current limits and BESS State-of-Charge (SoC) bounds [15]. When the BESS approaches its operational limits, aggressive inertial response can lead to DC-link voltage collapse or accelerated battery degradation.

Therefore, a critical research gap exists in developing an optimal, multivariable adaptive control framework for HRES that synergistically manages frequency regulation while strictly adhering to the physical constraints of the solar-wind-storage system.

To bridge this gap, this paper proposes a Linear Parameter-Varying Model Predictive Control (LPV-MPC) adaptive VSG framework for HRES. The primary contributions of this manuscript are threefold:

1. **Theoretical Formulation:** We develop an LPV state-space model that encapsulates the nonlinear coupling between VSG parameters (J, D) and grid frequency dynamics, enabling the application of advanced predictive control without the computational burden of nonlinear MPC.

2. **Constrained Optimization Framework:** We formulate a finite-horizon optimal control problem that dynamically synthesizes virtual inertia and damping. The cost function minimizes frequency deviation and RoCoF, while hard constraints explicitly enforce inverter apparent power limits, stochastic PV/wind generation availability, and BESS SoC dynamics.

3. **Numerical Validation:** Comprehensive numerical analysis on a modified IEEE 39-bus system validates the proposed MPC-VSG against conventional droop, fixed-VSG, and fuzzy-VSG methods under stochastic generation profiles and severe fault conditions.

Complementary optimization-theoretic tools include efficient extragradient methods for variational inequality problems with pseudo-monotone operators [16], adaptive numerical frameworks combining classical discretization with learning-based correction [17], and generalized convexity results underlying quadratic cost formulations [18]. The adaptive, data-efficient forecasting principles underlying the proposed LPV-MPC framework are further related to augmented reverse-training schemes for time-series forecasting [19] and to comparative studies of autoregressive and neural network models for forecasting volatile processes [20]. The underlying constrained QP formulation is further connected to robust optimization for quadratic programs under data uncertainty [21], simplified Lyapunov stability analysis for nonlinear systems [22], and viscosity-based iterative schemes for nonexpansive mappings [23].

The remainder of this paper is organized as follows: Section II details the theoretical formulation and HRES modeling. Section III presents the proposed LPV-MPC methodology. Section IV discusses the numerical results. Finally, Section V concludes the paper.

2. SYSTEM MODEL AND THEORETICAL FORMULATION

2.1. Hybrid System Architecture

The investigated HRES comprises a solar PV array, a Doubly Fed Induction Generator (DFIG) wind turbine interfaced via AC/DC converters, and a BESS connected via a bidirectional DC/DC converter. These sources share a common DC-link, which is interfaced to the AC grid via a three-phase GFM voltage source inverter (VSI) equipped with an LCL filter. The power balance at the DC-link is governed by:

$$C_{dc}V_{dc}\frac{dV_{dc}}{dt} = P_{PV} + P_{Wind} \pm P_{BESS} - P_{inv} \tag{1}$$

where C_{dc} is the DC-link capacitance, V_{dc} is the DC voltage, P_{PV} and P_{Wind} are the stochastic renewable powers, P_{BESS} is the battery power, and P_{inv} is the active power injected into the AC grid.

2.2. Virtual Synchronous Generator Dynamics

The GFM inverter is controlled to emulate the electromechanical dynamics of a synchronous machine. The core of the VSG is the swing equation:

$$J\frac{d\omega_m}{dt} = P_{ref} - P_e - D(\omega_m - \omega_g) \tag{2}$$

$$\frac{d\theta_m}{dt} = \omega_m \tag{3}$$

where J is the virtual moment of inertia, D is the damping factor, ω_m is the virtual angular frequency, ω_g is the grid frequency, P_{ref} is the reference active power, P_e is the measured electrical power, and θ_m is the virtual phase angle.

The reactive power control and voltage regulation are governed by a droop mechanism:

$$E_m = E_{ref} - K_q(Q_e - Q_{ref}) \tag{4}$$

where E_m is the virtual internal voltage magnitude, E_{ref} is the nominal voltage, K_q is the reactive power droop coefficient, and Q_e is the measured reactive power.

2.3. State-Space Formulation for Low-Inertia Grids

To apply optimal control, the nonlinear VSG dynamics must be formulated into a state-space representation. We define the state vector as $\mathbf{x} = [\Delta\delta, \Delta\omega_m, \Delta P_e, \Delta Q_e]^T$, where $\Delta\delta = \theta_m - \theta_g$ is the power angle. Small-signal linearization of the power flow equations yields:

$$\Delta P_e = \frac{E_m V_g}{X}\cos(\delta_0)\Delta\delta + \frac{V_g}{X}\sin(\delta_0)\Delta E_m \tag{5}$$

where V_g is the grid voltage and X is the equivalent line reactance.

The continuous-time state-space model is expressed as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c(J, D)\mathbf{x}(t) + \mathbf{B}_c(J)\mathbf{u}(t) + \mathbf{E}_c\mathbf{w}(t) \tag{6}$$

where the control input $\mathbf{u}(t) = [\Delta P_{ref}, \Delta E_{ref}]^T$, and the disturbance vector $\mathbf{w}(t) = \Delta\omega_g$. The system matrix $\mathbf{A}_c$ is highly dependent on the parameters J and D:

$$\mathbf{A}_c(J, D) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_P}{J} & -\frac{D}{J} & -\frac{1}{J} & 0 \\ K_P & 0 & -\omega_c & 0 \\ 0 & K_Q & 0 & -\omega_c \end{bmatrix} \tag{7}$$

where K_P and K_Q are the linearized power flow synchronizing coefficients, and ω_c is the cutoff frequency of the power measurement low-pass filter.

Because J and D appear in the denominator, the system is fundamentally nonlinear with respect to the adaptive parameters.

3. PROPOSED METHODOLOGY: LPV-MPC ADAPTIVE VSG

To dynamically optimize J and D without resorting to computationally prohibitive nonlinear MPC, we propose a Linear Parameter-Varying (LPV) formulation.

3.1. LPV Transformation and Discretization

We introduce a new virtual control vector $\mathbf{v}(t) = [\alpha(t), \beta(t)]^T$, where:

$$\alpha(t) = \frac{1}{J(t)}, \quad \beta(t) = \frac{D(t)}{J(t)} \tag{8}$$

This transformation renders the system matrix affine with respect to the scheduling parameters α and β. The continuous-time model is discretized using a zero-order hold with sampling time T_s, yielding the discrete LPV model:

$$\mathbf{x}_{k+1} = \mathbf{A}_d(\alpha_k, \beta_k)\mathbf{x}_k + \mathbf{B}_d(\alpha_k)\mathbf{u}_k + \mathbf{E}_d\mathbf{w}_k \tag{9}$$

3.2. Objective Function Formulation

The objective of the proposed MPC is to minimize frequency deviations ($\Delta\omega_m$) and RoCoF ($d\omega_m/dt$), penalize excessive control effort, and maintain the virtual parameters within bounds over a prediction horizon N_p and control horizon N_c. The quadratic cost function is defined as:

$$\begin{aligned} \mathcal{J}(\mathbf{x}_k, \mathbf{v}_k) = &\sum_{i=1}^{N_p}\left(\mathbf{x}_{k+i|k}^T\mathbf{Q}\mathbf{x}_{k+i|k} + \gamma\left|\frac{\Delta\omega_{m,k+i|k}}{T_s}\right|^2\right) \\ &+ \sum_{j=0}^{N_c-1}\Delta\mathbf{v}_{k+j|k}^T\mathbf{R}\Delta\mathbf{v}_{k+j|k} \end{aligned} \tag{10}$$

where $\mathbf{Q} \succeq 0$ and $\mathbf{R} \succ 0$ are weighting matrices, and γ is the penalization factor for RoCoF.

3.3. Multivariable Constraints

The optimization is subjected to strict physical constraints dictated by the HRES hardware.

1) Inverter Capacity Limits: The apparent power must not exceed the inverter rating S_{max}:

$$P_{e,k}^2 + Q_{e,k}^2 \le S_{max}^2 \tag{11}$$

To maintain convexity, this is linearized using a polygonal approximation.

2) BESS SoC Dynamics: The BESS absorbs or injects the transient power required to synthesize inertia. The SoC is updated as:

$$SoC_{k+1} = SoC_k - \frac{\eta T_s}{E_{cap}} P_{BESS,k} \tag{12}$$

where E_{cap} is the battery capacity and η is the efficiency. The SoC must remain within safe operational limits:

$$SoC_{min} \leq SoC_{k+i|k} \leq SoC_{max} \tag{13}$$

3) Adaptive Parameter Bounds: The virtual inertia is constrained by the available kinetic energy buffer in the BESS and the instantaneous headroom of the PV-Wind system:

$$J_{min} \leq J_k \leq J_{max}(SoC_k, P_{PV}, P_{Wind}) \tag{14}$$

where J_{max} is dynamically derated if the SoC approaches SoC_{min} or if the renewable generation is at maximum power point tracking (MPPT) limits. Consequently, the bounds on the LPV parameters are:

$$\alpha_{min,k} \leq \alpha_k \leq \alpha_{max}, \quad \beta_{min} \leq \beta_k \leq \beta_{max} \tag{15}$$

3.4. Optimization Solution

The resulting optimization problem is a constrained Quadratic Program (QP) solved at each sampling instant k:

$$\min_{\mathbf{v}} \mathcal{J}(\mathbf{x}_k, \mathbf{v}_k) \tag{16}$$

subject to equations (9) through (15). The first element of the optimal sequence $\mathbf{v}^*_{k|k}$ is extracted, inverted to find the optimal J_k and D_k, and applied to the VSG controller.

4. NUMERICAL RESULTS AND DISCUSSION

To evaluate the proposed LPV-MPC adaptive VSG, the framework is implemented in MATLAB/Simulink and tested on a modified IEEE 39-bus New England system. The system is modified to represent an ultra-low inertia grid by replacing 60% of the synchronous generation capacity with GFM HRES units.

4.1. Test System Parameters

The parameters of the HRES and the MPC controller are detailed in Table 1. The proposed method is benchmarked against three established control strategies: (1) Standard Droop Control, (2) Fixed-Parameter VSG ($J = 2.0$ kg·m^2, $D = 15$ N·m·s/rad), and (3) Fuzzy-Logic Adaptive VSG.

Table 1. System and controller parameters.

Symbol	Parameter Description	Value
S_{max}	Inverter Nominal Rating	2.5 MVA
V_g	Nominal Grid AC Voltage (L-L)	690 V
f_n	Nominal Grid Frequency	50 Hz
V_{dc}	Nominal DC-Link Voltage	1200 V
E_{cap}	BESS Energy Capacity	1.5 MWh
SoC_{bounds}	BESS Safe Operating Range	$[0.2, 0.9]$ p.u.
T_s	MPC Sampling Time	5 ms
N_p, N_c	Prediction and Control Horizons	$15, 3$
J_{bounds}	Virtual Inertia Limits	$[0.5, 4.5]$ kg·m^2
D_{bounds}	Damping Coefficient Limits	$[5, 40]$ N·m·s/rad

4.2. Scenario A: Severe Load Step under Low Inertia

At $t = 2.0$ s, a sudden active power load step of 0.15 p.u. is applied to the grid. Table 2 presents the comparative transient performance metrics.

The numerical results indicate that the standard droop controller suffers from an unacceptable RoCoF of 2.85 Hz/s, violating standard grid codes. The Fixed-Parameter VSG improves the response but still exhibits significant frequency nadir (49.18 Hz). The proposed LPV-MPC dynamically maximizes J during the initial transient to arrest the RoCoF (1.10 Hz/s, a 42% reduction compared to fixed VSG) and subsequently increases D to rapidly damp the oscillations, achieving a settling time of merely 1.9 seconds. Furthermore, the MPC explicitly prevents the BESS from violating discharge rate limits, a constraint ignored by the Fuzzy-Logic approach.

Table 2. Transient performance comparison (0.15 p.u. load step).

Control Strategy	Max Freq. Deviation (Hz)	Max RoCoF (Hz/s)	Settling Time (s)
Standard Droop	1.15	2.85	4.2
Fixed-Parameter VSG	0.82	1.90	3.5
Fuzzy-Logic VSG	0.65	1.45	2.8
Proposed LPV-MPC	**0.51**	**1.10**	**1.9**

Table 3. Robustness analysis under severe asymmetrical fault.

Control Strategy	CCT (ms)	Max PCC Voltage Sag	BESS Transient Energy (MJ)
Standard Droop	180	0.65 p.u.	1.2
Fixed-Parameter VSG	240	0.48 p.u.	4.5
Fuzzy-Logic VSG	295	0.35 p.u.	5.8
Proposed LPV-MPC	**360**	**0.22** p.u.	**3.1**

4.3. Scenario B: Asymmetrical Grid Fault and Robustness Analysis

To evaluate large-signal stability, a severe two-phase-to-ground fault is applied at a neighboring bus for 150 ms. The robustness of the controllers is assessed based on the Critical Clearing Time (CCT), maximum voltage sag at the Point of Common Coupling (PCC), and the transient energy consumed from the BESS during recovery (Table 3).

During the fault, the Fixed-Parameter VSG continues to inject active power based on its rigid swing equation, leading to severe power angle divergence and a relatively low CCT of 240 ms. The proposed LPV-MPC detects the grid voltage collapse and instantly minimizes the virtual inertia ($\alpha \to \alpha_{max}$), decoupling the inverter from the grid frequency drop and preventing pole slipping. Consequently, the CCT is extended to 360 ms. Moreover, by optimizing the control effort via the weighting matrix $\mathbf{R}$, the proposed MPC utilizes only 3.1 MJ of BESS energy during recovery, significantly less than the 5.8 MJ consumed by the unconstrained Fuzzy-Logic VSG, thereby preserving battery lifespan.

5. CONCLUSION

This paper has presented a Linear Parameter-Varying Model Predictive Control (LPV-MPC) framework for adaptive Virtual Synchronous Generator (VSG) operation in hybrid solar-wind-storage systems. By translating the nonlinear, parameter-dependent VSG dynamics into an affine LPV model, the proposed methodology enables real-time, optimal synthesis of virtual inertia and damping via constrained Quadratic Programming. Unlike heuristic adaptive methods, the proposed MPC explicitly incorporates the physical constraints of the inverter and the BESS State-of-Charge, ensuring that the provision of synthetic inertia does not compromise hardware integrity. Numerical simulations on a modified IEEE 39-bus ultra-low inertia system demonstrated that the proposed framework outperforms conventional droop, fixed VSG, and fuzzy-logic VSG controllers. Specifically, it reduced the maximum RoCoF by 42% and improved the Critical Clearing Time during asymmetrical faults by 50% compared to fixed-parameter VSG, while strictly bounding BESS transient energy consumption. Future work will extend this framework to decentralized, cooperative MPC architectures for multi-microgrid clusters and validate the algorithms using hardware-in-the-loop (HIL) testbeds.

ACKNOWLEDGEMENTS

[Author acknowledgements placeholder.]

REFERENCES

[1] J. M. Guerrero, J. C. Vasquez, J. Matas, L. G. de Vicuna, and M. Castilla, "Hierarchical Control of Droop-Controlled AC and DC Microgrids—A General Approach Toward Standardization," *IEEE Trans. Ind. Electron.*, vol. 58, no. 1, pp. 158–172, Jan. 2011. DOI: 10.1109/TIE.2010.2066534.

[2] J. Rocabert, A. Luna, F. Blaabjerg, and P. Rodriguez, "Control of Power Converters in AC Microgrids," *IEEE Trans. Power Electron.*, vol. 27, no. 11, pp. 4734–4749, Nov. 2012. DOI: 10.1109/TPEL.2012.2199334.

[3] Q. Zhong and G. Weiss, "Synchronverters: Inverters That Mimic Synchronous Generators," *IEEE Trans. Ind. Electron.*, vol. 58, no. 4, pp. 1259–1267, April 2011. DOI: 10.1109/TIE.2010.2048839.

[4] H. Bevrani, T. Ise, and Y. Miura, "Virtual synchronous generators: A dynamic and control perspective," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1929–1937, July 2014. DOI: 10.1109/TPWRS.2013.2281162.

[5] B. Kroposki et al., "Achieving a 100% Renewable Grid: Operating Electric Power Systems with Extremely High Levels of Variable Renewable Energy," *IEEE Power and Energy Magazine*, vol. 15, no. 2, pp. 61–73, March-April 2017. DOI: 10.1109/MPE.2016.2637122.

[6] J. Alipoor, Y. Miura, and T. Ise, "Power Routing for Cluster of Microgrids with Virtual Synchronous Generator Control," *IEEE Trans. Power Electron.*, vol. 30, no. 5, pp. 2883–2893, May 2015. DOI: 10.1109/TPEL.2014.2362541.

[7] S. Vazquez, J. I. Leon, L. G. Franquelo, J. Rodriguez, H. A. Young, A. Marquez, and P. Zanchetta, "Model Predictive Control: A Review of Its Applications in Power Electronics," *IEEE Ind. Electron. Mag.*, vol. 8, no. 1, pp. 16–31, March 2014. DOI: 10.1109/MIE.2013.2290138.

[8] R. Majumder, "Some Aspects of Stability in Microgrids," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3243–3252, Aug. 2013. DOI: 10.1109/TPWRS.2012.2234146.

[9] W. Zhang, A. M. Cantarellas, J. Rocabert, A. Luna, and P. Rodriguez, "Synchronous Power Controller with Flexible Droop Characteristics for Renewable Power Generation Systems," *IEEE Trans. Sustain. Energy*, vol. 7, no. 4, pp. 1572–1582, Oct. 2016. DOI: 10.1109/TSTE.2016.2569274.

[10] S. D'Arco and J. A. Suul, "Equivalence of Virtual Synchronous Machines and Frequency-Droops for Converter-Based MicroGrids," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 394–395, Jan. 2014. DOI: 10.1109/TSG.2013.2288000.

[11] U. Markovic, O. Stanojev, P. Aristidou, E. Vrettos, D. Callaway, and G. Hug, "Understanding Small-Signal Stability of Low-Inertia Systems," *IEEE Trans. Power Syst.*, vol. 36, no. 2, pp. 1490–1502, March 2021. DOI: 10.1109/TPWRS.2020.3015408.

[12] T. Kerdphol, F. S. Rahman, Y. Mitani, M. Watanabe, and S. Küäm, "Virtual Inertia Control Application to Enhance Frequency Stability of Interconnected Power Systems with High Renewable Energy Penetration," *IEEE Trans. Ind. Appl.*, vol. 54, no. 1, pp. 681–693, Jan.-Feb. 2018. DOI: 10.1109/TIA.2017.2785084.

[13] U. Markovic, Z. Chu, P. Aristidou, and G. Hug, "LQR-Based Adaptive Virtual Synchronous Machine for Power Systems With High Inverter Penetration," *IEEE Trans. Sustain. Energy*, vol. 10, no. 3, pp. 1501–1512, July 2019. DOI: 10.1109/TSTE.2018.2887147.

[14] J. Fang, H. Li, Y. Tang, and F. Blaabjerg, "Virtual Inertia Management for Microgrids," *IEEE Trans. Ind. Inform.*, vol. 14, no. 11, pp. 4885–4896, Nov. 2018. DOI: 10.1109/TII.2018.2838382.

[15] A. Tayyebi, D. Groß, A. Anta, F. Küpz, and F. Dörfler, "Frequency Stability of Synchronous Machines and Grid-Forming Power Converters," *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 8, no. 2, pp. 1004–1018, June 2020. DOI: 10.1109/JESTPE.2019.2893569.

[16] M. L. Abdullahi, O. Wahab, S. A. Aderoju, G. N. Mohammed, D. John, and S. A. Musa, "An efficient extragradient method for solving variational inequality problems with pseudo-monotone operators," *Annals of Mathematics and Computer Science*, vol. 32, 2026. DOI: 10.56947/amcs.v32.701.

[17] B. Padhy, P. S. Mahapatra, and T. Samantara, "Adaptive Hybrid B-Spline PINN for Fractional Equations," *Annals of Mathematics and Computer Science*, vol. 35, 2026. DOI: 10.56947/amcs.v35.864.

[18] J. El-achky, "Some Results on Symmetrized A H-Convexity," *Annals of Mathematics and Computer Science*, vol. 35, 2026. DOI: 10.56947/amcs.v35.860.

[19] F. Kamalov, I. Gurrib, L. Smail, and Z. El Khatib, "ART-LSTM: Augmented Reverse Training for Data-Efficient Time Series Forecasting," *Journal of Information & Knowledge Management*, vol. 24, no. 6, art. 2550062, 2025. DOI: 10.1142/S0219649225500625.

[20] F. Kamalov, I. Gurrib, S. Moussa, and A. Nazir, "A Comparative Study of Autoregressive and Neural Network Models: Forecasting the GARCH Process," in *Proc. Int. Conf. Intelligent Computing (ICIC)*, LNCS vol. 13395, Springer, Cham, 2022, pp. 589–603. DOI: 10.1007/978-3-031-13832-4_48.

[21] S. Sanogo, M. Zongo, M. Barro, and S. Traore, "Robust optimization for quadratic programs with data uncertainty using the concept of stability radius," *Gulf Journal of Mathematics*, vol. 16, no. 2, pp. 266–277, 2024. DOI: 10.56947/gjom.v16i2.1823.

[22] S. Azioune, W. Laouira, M. Berkal, E. M. Elsayed, and M. B. Almatrafi, "Simplified Lyapunov Stability for Chaotic Systems," *Gulf Journal of Mathematics*, vol. 23, no. 2, 2026. DOI: 10.56947/w0sqw731.

[23] Y. Vaishnav, V. K. Sahu, and R. Tiwari, "A viscosity implicit midpoint rule for quasi-nonexpansive mappings," *Gulf Journal of Mathematics*, vol. 23, no. 1, pp. 1–14, 2026. DOI: 10.56947/jb97dk92.