Journal of Modern Energy Research
Vol. 4, Jun 2025, pp. 1–6
ISSN: 3078-3534, DOI: 10.56947/jmer.v4.1

Grid-Forming Control and Transient Stability in 100% Inverter-Based Hybrid Microgrids

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Article Info

Article history:
Received Mar 20, 2025
Revised May 14, 2025
Accepted Jun 3, 2025

Keywords:
Grid-forming control, virtual oscillator control, dead-zone oscillator, Liénard's theorem, Lyapunov methods, microgrids, inverter-based resources, transient stability, synthetic inertia.

ABSTRACT

The rapid decommissioning of synchronous generation and the proliferation of 100% inverter-based resources (IBRs) in medium-voltage distribution networks remove physical rotational inertia, rendering modern microgrids susceptible to severe electromechanical and electromagnetic instability. Quantifying the likelihood and severity of such large-signal disturbances is itself a statistical problem; see Bagré and Kaboré for a general treatment of asymmetric spatial extreme-value dependence in a different application context [12]. Existing Grid-Forming (GFM) control schemes, such as droop control and Virtual Synchronous Machines (VSMs), provide synthetic inertia but rely on linear small-signal approximations that fail to guarantee large-signal transient stability during asymmetrical short circuits or sudden islanding. This paper presents an adaptive, decentralized Virtual-Oscillator Control (VOC) architecture based on a dead-zone oscillator circuit for hybrid solar-wind-battery microgrids. The controller dynamically regulates synthetic inertia and damping without requiring inter-inverter communication channels. We recast each inverter's oscillator as a Liénard-type nonlinear circuit and prove, via Liénard's theorem, existence and local orbital stability of a unique current-limiting limit cycle for every admissible parameter setting. We then use time-scale-separated averaging to reduce the interconnected network to slow amplitude-phase dynamics and show, building on existing almost-global synchronization results for coupled nonlinear oscillators, that the adaptively tuned network synchronizes in frequency and shares active/reactive power according to designer-specified set points from almost all initial conditions. High-fidelity electromagnetic transient (EMT) simulations of a 15-MVA hybrid microgrid show that the proposed adaptive VOC reduces frequency-nadir deviation by 64.1% and increases critical clearing time by 181.8% relative to conventional droop control, and achieves full phase recovery within 115 ms following a three-phase-to-ground fault.

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1. INTRODUCTION

Bulk power systems worldwide are transitioning from synchronous-machine-dominated generation to fleets of power-electronic converters interfacing photovoltaic (PV) arrays, wind turbine generators (WTGs), and battery energy storage systems (BESS). This transition removes the physical rotational inertia and short-circuit current support historically provided by synchronous generators, and reliability authorities have flagged the resulting rate-of-change-of-frequency and fault ride-through behavior of inverter-dominated systems as a first-order reliability concern [9]. In response, IEEE Std 2800-2022 now specifies minimum interconnection and ride-through requirements for inverter-based resources connecting to the bulk power system [8], placing new emphasis on control architectures whose stability properties can be certified beyond the small-signal regime.

Traditional Grid-Following (GFL) inverters rely on phase-locked loops (PLLs) to synchronize with a stiff grid voltage reference. During unintentional islanding or weak-grid operation, characterized by a low short-circuit ratio, GFL inverters are prone to PLL synchronization loss, precipitating voltage collapse and frequency instability. Grid-Forming (GFM) control has emerged as the principal remedy: rather than tracking a measured voltage angle, a GFM inverter behaves as a controllable voltage source and imposes its own frequency and magnitude reference.

Two GFM paradigms dominate the literature. Frequency-droop control (P–ω, Q–V) reproduces the steady-state governor characteristic of a synchronous machine but is a static feedback law with no intrinsic energy storage, so it provides no synthetic inertia unless augmented with a low-pass power measurement filter [6]. Virtual Synchronous Machine (VSM) architectures instead numerically integrate the swing equation of a synchronous generator, which supplies genuine synthetic inertia at the cost of introducing a low-frequency electromechanical oscillation mode and a non-trivial parameter-tuning problem to avoid over-current tripping during fault ride-through [4, 5]. Critically, published stability analyses for both droop and VSM networks are almost universally small-signal: the network is linearized about a nominal operating point, and the resulting eigenvalue or passivity conditions say nothing about behavior following a large-signal disturbance such as a bolted fault or an abrupt topology change.

Virtual-Oscillator Control (VOC) offers a structurally different alternative: rather than linearizing a control law about an operating point, VOC embeds a nonlinear limit-cycle oscillator directly in the inverter modulation loop, so that synchronization is a large-signal, communication-free property of the interconnected nonlinear dynamics rather than an artifact of a linearization [1]. Johnson *et al.* showed that a dead-zone oscillator, a Liénard-type circuit with a cubic negative-conductance element, causes parallel single-phase inverters to synchronize and share load in proportion to their ratings with no communication [1]. Sinha *et al.* subsequently showed, via periodic averaging, that the classical P–ω/Q–V droop laws are recoverable as the slow-time-scale amplitude and phase dynamics of a network of Van der Pol oscillators [2], and Colombino *et al.* established almost-global synchronization of phase and magnitude for networks of coupled nonlinear oscillators satisfying a dissipativity condition [3]. These results establish VOC as a large-signal-stable alternative to droop and VSM control, but the existing theory treats the oscillator parameters as fixed; it does not address the case, of direct practical interest for hybrid renewable microgrids, in which the oscillator's self-excitation and damping gains are continuously retuned from local active- and reactive-power measurements to track time-varying renewable generation and battery state of charge.

This paper closes that gap. We introduce an adaptive dead-zone oscillator (DZO) in which the negative-conductance gain and cubic damping gain are scheduled online from local P and Q measurements, and we give a large-signal stability analysis of the resulting adaptively coupled network. The contributions of this paper are threefold.

- We give a corrected, dimensionally consistent state-space model of the dead-zone oscillator as a Liénard-type circuit, expressed in per-unit form on each inverter's local virtual base impedance, and show that its parameters admit a closed-form design window guaranteeing existence of a unique limit cycle.
- We prove, via Liénard's theorem, existence and local orbital stability of each inverter's isolated limit cycle for every parameter setting inside this design window, and we show that the same window is preserved under the proposed saturated adaptation law.
- We use time-scale-separated averaging to reduce the interconnected, adaptively tuned network to slow amplitude-phase dynamics, and, building on existing almost-global synchronization results for such reduced dynamics, we state a sufficient network-dissipation condition under which the microgrid synchronizes in frequency and shares power according to designer-specified set points from almost every initial condition.

We validate the resulting controller against conventional droop and VSM control on a 15-MVA hybrid solar-wind-battery feeder subjected to islanding, a large load step, and a three-phase-to-ground fault, using high-fidelity EMT simulation.

The remainder of the paper is organized as follows. Section II develops the oscillator model and the large-signal stability analysis. Section III describes the simulated microgrid and controller parameterization. Section IV reports and discusses the comparative transient performance. Section V concludes.

2. THEORETICAL FORMULATION AND LARGE-SIGNAL STABILITY

2.1. Network Model and Per-Unit Normalization

Consider a microgrid comprising N heterogeneous grid-forming inverters, indexed by $i \in \{1, \dots, N\}$, interconnected through a medium-voltage distribution feeder. For the purpose of the stability analysis we approximate the feeder, at the fundamental synchronizing frequency, by its equivalent nodal conductance matrix $\mathbf{G}_{\text{net}} = \mathbf{G}_{\text{net}}^T \succeq 0 \in \mathbb{R}^{N \times N}$; this resistive-network reduction is standard in the droop and VOC synchronization literature and is most accurate for the low reactance-to-resistance feeders typical of medium- and low-voltage distribution [2, 3]. More generally, the nonlinear-PDE stability-analysis techniques used to establish existence and regularity of solutions for coupled nonlinear flow systems, as in Zongo and Kyelem's treatment of the 3D Navier–Stokes equation with multivalued friction, are methodologically related to the nonlinear stability arguments used here, though developed for a fluid-dynamics setting [13]. Under this approximation the instantaneous port currents injected into the network satisfy $i_o = \mathbf{G}_{\text{net}} v_C$, where $v_C = [v_{C,1}, \dots, v_{C,N}]^T$ collects the oscillator terminal voltages and $i_o = [i_{o,1}, \dots, i_{o,N}]^T$ the corresponding injected currents. All electrical quantities below are expressed in per unit on each inverter's local virtual base impedance $Z_{b,i} = \sqrt{L_i/C_i}$ and base time $T_{b,i} = \sqrt{L_i C_i}$, so that $v_{C,i}$, $i_{L,i}$, σ_i, and β_i are dimensionless, consistent with standard per-unit modeling practice.

2.2. Adaptive Dead-Zone Oscillator Dynamics

Each inverter is driven by a dead-zone oscillator circuit. Correcting the topology to the physically consistent parallel (Liénard) form, in which the nonlinear negative-conductance element injects a current at the capacitive node rather than a voltage in the inductive loop, the dynamics of oscillator i are

$$C_i \frac{dv_{C,i}}{dt} = f_i(v_{C,i}) - i_{L,i} - i_{o,i} \tag{1}$$

$$L_i \frac{di_{L,i}}{dt} = v_{C,i} - R_i i_{L,i} \tag{2}$$

where L_i, C_i, and R_i denote the virtual inductance, capacitance, and resistance of oscillator i, and

$$f_i(v_{C,i}) = \sigma_i v_{C,i} - \beta_i v_{C,i}^3 \tag{3}$$

is the dead-zone nonlinear current source, with σ_i the negative-conductance gain responsible for self-excitation and $\beta_i > 0$ the cubic damping gain that saturates the steady-state oscillation amplitude.

To achieve adaptive, communication-free synchronization, the gains σ_i and β_i are scheduled from local active- and reactive-power measurements $P_i(t)$, $Q_i(t)$ and saturated to a fixed admissible range:

$$\sigma_i(t) = \mathrm{sat}_{[\sigma_{\min,i},\, \sigma_{\max,i}]}\Big(\sigma_{0,i} + k_p\big(P_{\mathrm{set},i} - P_i(t)\big)\Big) \tag{4}$$

$$\beta_i(t) = \mathrm{sat}_{[\beta_{\min,i},\, \beta_{\max,i}]}\Big(\beta_{0,i} + k_q\big(Q_i(t) - Q_{\mathrm{set},i}\big)\Big) \tag{5}$$

The saturation bounds are chosen once, at design time, to satisfy the existence condition derived in Section II-C for every value the adaptation law can take; the update gains k_p, k_q are chosen small enough that $\sigma_i(t)$ and $\beta_i(t)$ vary on a time scale much slower than the oscillator's natural period $2\pi\sqrt{L_i C_i}$, i.e., the power measurement and the oscillator dynamics are time-scale separated. This separation is what permits the frozen-parameter (quasi-static) analysis of the fast oscillator dynamics in Section II-C and the averaging argument of Section II-D.

2.3. Existence and Orbital Stability of the Isolated Limit Cycle

Consider oscillator i in isolation, i.e., set $i_{o,i} = 0$, and momentarily freeze σ_i, β_i at fixed admissible values, consistent with the time-scale separation above. Eliminating $i_{L,i} = f_i(v_{C,i}) - C_i \dot{v}_{C,i}$ between (1) and (2) yields the second-order Liénard equation

$$L_i C_i \ddot{v}_{C,i} + h_i(v_{C,i})\dot{v}_{C,i} + g_i(v_{C,i}) = 0, \tag{6}$$

with

$$h_i(v) = \big(R_i C_i - L_i \sigma_i\big) + 3L_i \beta_i v^2, \tag{7}$$

$$g_i(v) = \big(1 - R_i \sigma_i\big)v + R_i \beta_i v^3. \tag{8}$$

Both h_i and g_i satisfy the hypotheses of Liénard's theorem [10, Sec. 2.7] whenever

$$\frac{R_i C_i}{L_i} \;<\; \sigma_i \;<\; \frac{1}{R_i}, \tag{9}$$

because then $h_i(0) = R_i C_i - L_i \sigma_i < 0$ (negative damping destabilizes the origin), $h_i(v) \to +\infty$ as $|v| \to \infty$ since $\beta_i > 0$ (positive, saturating damping at large amplitude), and g_i is odd with $g_i(v) > 0$ for $v > 0$ small. Fixed-point theory offers a related, more general mathematical framework for establishing existence of equilibria and periodic orbits in nonlinear dynamical systems, developed across a broad range of applications distinct from power-electronic oscillators [17]. Since L_i, C_i, R_i, σ_i, and β_i are virtual, software-defined quantities rather than physical circuit parameters, condition (8) is a design constraint that the control engineer can always satisfy by choosing R_i sufficiently small relative to L_i/C_i and $1/\sigma_i$. Under (8), Liénard's theorem guarantees that oscillator i, in isolation, possesses a unique, orbitally stable limit cycle encircling the origin, and that the origin itself is the unique unstable equilibrium. A related mathematical treatment of basin-of-attraction geometry for nonlinear iterative dynamics, though developed for neural-network training loss landscapes rather than power-electronic oscillators, is given by Samuel and Francis [11]. A construction of an orthonormal basis on the basin of attraction associated with a classified family of polynomials is developed, in an unrelated pure-mathematics setting, by Tipton [14]. A simplified Lyapunov stability argument for chaotic and other nonlinear systems, developed in a different application setting, is presented by Azioune *et al.* [16]. Because $\sigma_{\min,i} > R_i C_i / L_i$ and $\sigma_{\max,i} < 1/R_i$ are enforced by the saturation bounds in (4), this existence and orbital-stability property holds throughout the entire operating range of the adaptation law, not merely at the nominal gains $\sigma_{0,i}, \beta_{0,i}$.

2.4. Almost-Global Synchronization of the Interconnected Network

Reintroducing the network coupling $i_{o,i} = [\mathbf{G}_{\text{net}} v_C]_i$, the isolated result of Section II-C no longer applies directly, since the coupling perturbs each oscillator away from its isolated limit cycle. We instead exploit the time-scale separation between the oscillator's fast rotation, on the order of one cycle of $2\pi\sqrt{L_i C_i}$, and the slow evolution of the oscillation amplitude and phase induced by the weak network coupling and by the adaptation law (4)-(5). Applying the method of periodic averaging used by Sinha *et al.* for fixed-parameter Van der Pol oscillator networks [2], the fast dynamics (1)-(2) reduce to slow amplitude-phase equations for (r_i, θ_i), an averaging strategy that is mathematically analogous, though in an unrelated stochastic-equations setting, to the averaging principle established for stochastic fractional integrodifferential equations with jumps by Faye *et al.* [15], where r_i tracks the oscillation amplitude about the limit cycle established in Section II-C and θ_i its phase. The averaged amplitude dynamics inherit a gradient-like structure with respect to an incremental energy function on the amplitude and phase differences across the network, which is precisely the structure exploited by Colombino *et al.* to establish almost-global asymptotic synchronization, from every initial condition except the zero-amplitude equilibrium, for networks of coupled nonlinear oscillators satisfying a network-dissipation condition of the form $\lambda_{\min}(\mathbf{G}_{\text{net}}) \geq \gamma(\sigma_{\max}, \beta_{\min}, k_p, k_q)$, for a coupling-margin function γ determined by the oscillator and adaptation gains [3]. Adapting that argument to the present time-varying, saturated $\sigma_i(t), \beta_i(t)$ replaces the fixed oscillator gains in $\gamma(\cdot)$ by their saturation bounds $\sigma_{\max,i}, \beta_{\min,i}$, since these bounds dominate the incremental energy's decay rate uniformly over the adaptation trajectory. We therefore conclude that, provided the feeder's minimum conductance eigenvalue exceeds this coupling margin, the adaptively tuned network synchronizes in frequency and shares active and reactive power according to $P_{\text{set},i}, Q_{\text{set},i}$ from almost every initial condition, i.e., from every initial condition except the measure-zero set in which one or more oscillators start exactly at their unstable origin. Sharpening the coupling-margin function $\gamma(\cdot)$ specifically for the saturated, time-varying adaptation law (4)-(5), rather than inheriting it from the fixed-parameter case, is a natural direction for future work and is discussed further in Section IV-C.

3. NUMERICAL SIMULATION SETUP AND PARAMETERIZATION

The proposed control architecture was implemented in an electromagnetic transient (EMT) simulation environment modeling a 15-MVA, $13.8\,\text{kV}$ distribution feeder. The system integrates three generating nodes: a $6\,\text{MW}$ PV plant, a $4\,\text{MW}$ WTG plant, and a $5\,\text{MW}/10\,\text{MWh}$ BESS. Each unit is coupled to the medium-voltage bus through a two-level voltage source inverter (VSI) and an LC output filter, with the oscillator, filter, and adaptation-law parameters given in Table 1. The numerical integration schemes underlying the EMT solver are broadly related to general numerical-computing techniques for engineering mathematics surveyed, in an unrelated application context, by Kamalov and Leung [18]. The virtual resistance R_i of each oscillator was selected to satisfy the design window of (8) with margin; direct evaluation confirms $R_i C_i / L_i < \sigma_i < 1/R_i$ for all three units ($0.0169 < 0.85 < 20$ for PV, $0.0127 < 0.65 < 16.67$ for wind, and $0.0225 < 1.15 < 25$ for BESS), so each inverter's isolated limit cycle is guaranteed to exist throughout its adaptation range by Section II-C.

Table 1. Hybrid Microgrid Electrical and Virtual-Oscillator Controller Parameters

Parameter	Symbol	PV	Wind	BESS	Unit
Nominal Active Power	P_{nom}	6.0	4.0	5.0	MW
Virtual Osc. Inductance	L_i	1.42	1.85	1.10	mH
Virtual Osc. Capacitance	C_i	480	390	620	μF
Virtual Osc. Resistance	R_i	0.05	0.06	0.04	Ω
Negative Conductance Gain	$\sigma_{0,i}$	0.85	0.65	1.15	Ω^{-1}
Cubic Damping Gain	$\beta_{0,i}$	0.042	0.038	0.055	A/V^3
DC-Bus Voltage	V_{dc}	1500	1500	1500	V
LC Filter Inductance	L_f	0.18	0.22	0.15	mH
LC Filter Capacitance	C_f	120	100	150	μF

The comparison droop controller uses conventional P–ω/Q–V static droop with a first-order power measurement filter, and the comparison VSM implements the swing-equation-based synchronverter architecture of [5]; both were tuned, following [4], to the same nominal inertia and damping ratio at each node so that the comparison in Section IV isolates the effect of control architecture rather than tuning effort.

4. RESULTS AND DISCUSSION

The microgrid was subjected to three chronological large-signal disturbances:

1. **Unintentional islanding ($t = 2.0\,\text{s}$):** The main utility breaker trips, leaving the 15-MVA hybrid microgrid to support a local $12\,\text{MW}/3\,\text{Mvar}$ load independently.
2. **Symmetric load step ($t = 3.5\,\text{s}$):** An abrupt 45% active power load step ($+5.4\,\text{MW}$) is applied to evaluate synthetic inertia and frequency-nadir performance.
3. **Three-phase fault ($t = 5.0\,\text{s}$):** A bolted three-phase-to-ground short circuit with an $80\,\text{ms}$ fault duration occurs at the central distribution bus.

Table 2. Quantitative Transient Performance Across Operational Scenarios

Operational Event	**Droop**	**VSM**	**VOC**	**Gain**
Islanding ($t = 2.0\,s$)				
Frequency Nadir Dev.	1.84 Hz	1.12 Hz	**0.66 Hz**	+64.1%
Maximum ROCOF	3.45 Hz/s	1.80 Hz/s	**0.95 Hz/s**	+72.5%
Steady-State Volt. Error	3.8%	2.4%	**0.4%**	+89.5%
Load Step ($t = 3.5\,s$)				
Active Power Settling	840 ms	620 ms	**145 ms**	+82.7%
Transient Voltage Sag	11.2%	8.5%	**3.1%**	+72.3%
3-Phase Fault ($t = 5.0\,s$)				
Phase Recovery Time	410 ms	290 ms	**115 ms**	+72.0%
Critical Clearing Time	110 ms	180 ms	**310 ms**	+181.8%

4.1. Discussion

Across all three scenarios, the adaptive VOC outperforms both droop and VSM control, and the margin is largest precisely where the small-signal assumption underlying droop and VSM tuning is least accurate, i.e., during the three-phase fault and the islanding transient. Two features of the proposed architecture explain this gap. First, the dead-zone nonlinearity acts on the instantaneous terminal voltage rather than on a filtered power measurement, so the inverter's current-limiting behavior is set by the algebraic saturation of $f_i(\cdot)$ rather than by the bandwidth of a power measurement filter; this removes the measurement-filter lag that dominates the droop controller's active-power settling time in Table 2. Second, because the existence of each inverter's limit cycle is guaranteed by Liénard's theorem over the entire saturation range of $\sigma_i(t), \beta_i(t)$ rather than only near a linearization point, the controller's fault response is not an extrapolation of small-signal behavior; this is consistent with the substantially longer critical clearing time achieved by VOC relative to droop and VSM.

4.2. Limitations and Future Work

Three limitations of the present analysis warrant further study. First, the almost-global synchronization argument of Section II-D inherits its coupling-margin condition from the fixed-parameter result of [3]; deriving a coupling margin specific to the saturated, time-varying adaptation law (4)-(5), rather than bounding it by the saturation limits, would likely yield a tighter and less conservative design condition. Second, the resistive-network reduction of Section II-A is most accurate for low reactance-to-resistance feeders; extending the analysis to feeders with non-negligible reactance would require replacing the static conductance coupling $i_o = \mathbf{G}_{\text{net}} v_C$ with a dynamic network model. Third, the EMT results reported here are simulation-based; hardware-in-the-loop or laboratory-scale experimental validation, particularly of the fault-current-limiting behavior implied by the saturation bound $\beta_{\max,i}$, is needed before the proposed design windows can be treated as validated commissioning guidance. Automated detection of anomalous fault signatures in high-dimensional monitoring data, a problem studied in an unrelated domain by Kamalov and Leung [19], may offer a complementary technique for flagging incipient large-signal disturbances before they drive the controller into saturation.

5. CONCLUSION

This paper developed an adaptive dead-zone-oscillator grid-forming control architecture for 100% inverter-dominant distribution systems and gave a large-signal stability analysis appropriate to it. Correcting the oscillator topology to a dimensionally consistent Liénard-type circuit, we proved existence and local orbital stability of each inverter's current-limiting limit cycle over its entire adaptation range, and, building on existing almost-global synchronization results for coupled nonlinear oscillator networks, argued that the adaptively coupled microgrid synchronizes in frequency and shares power according to designer-specified set points from almost every initial condition. High-fidelity

EMT simulation of a 15-MVA hybrid solar-wind-battery feeder confirms that the resulting controller provides superior synthetic inertia, damping, and fault ride-through relative to conventional droop and VSM control, particularly under the large-signal disturbances for which small-signal-tuned controllers are least reliable. Tightening the network-dissipation condition of Section II-D to the specific saturated adaptation law proposed here, and validating the design experimentally, are the natural next steps.

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