

Physics-Constrained Diagnostic Framework for Predictive Thermal Runaway and Degradation in Grid-Scale Battery Storage

Evgeny Mikhailov¹, Ivan Novikov²

¹Department of Electrical Engineering, Yaroslavl State University, Yaroslavl, Russia

²Department of Computer and Electrical Engineering, North Caucasus Federal University, Stavropol, Russia

Article Info

Article history:

Received Apr 1, 2025

Revised May 21, 2025

Accepted Jun 15, 2025

Keywords:

Battery management systems, battery diagnostics, thermal runaway, internal short circuit, solid-electrolyte interphase, lithium plating, physics-informed observers, remaining useful life, grid-scale energy storage.

ABSTRACT

Utility-scale lithium-ion battery energy storage systems (BESS) deployed for dynamic frequency regulation and peak shaving experience nonlinear electrochemical aging and elevated risk of internal short circuits (ISCs). Purely data-driven diagnostic models fail to generalize across battery chemistries and ambient conditions, while high-fidelity electrochemical models exceed the computational budget of real-time battery management systems (BMS). This paper introduces an explainable, physics-constrained diagnostic framework that couples solid-electrolyte interphase (SEI) growth kinetics and lithium-plating physics with a real-time thermal residual observer. The observer propagates the coupled aging and thermal physics on the slow (hours-to-days) time scale associated with electrochemical degradation and flags fast (seconds-to-minutes), physically inconsistent deviations as candidate ISC events using a sequential residual test, which structurally decouples reversible thermal transients and expected capacity fade from irreversible short-circuit heating. Validated using telemetry from a 50-MWh utility BESS over 3,000 equivalent full cycles (EFCs), the physics-constrained approach identifies ISC onset 41.8 minutes earlier than industry-standard threshold alarms with a 0.3% false alarm rate, while forecasting remaining useful life (RUL) with a mean absolute percentage error (MAPE) of 1.6%–2.3%. Parametric lifetime-distribution fitting of the kind used to summarize RUL statistics is related, in a different application setting, to the four-parameter lifetime model recently proposed by Kahloul and Yahia [22].

This is an open access article under the CC BY-SA license.



Corresponding Author:

Evgeny Mikhailov
Department of Electrical Engineering
Yaroslavl State University
Yaroslavl, Russia
Email: evgeny.mikhailov@uniyar.ac.ru

1. INTRODUCTION

Utility-scale lithium-ion BESS increasingly perform dual duty as both energy-arbitrage and ancillary-service assets, most notably fast frequency regulation, where sub-minute bidirectional current cycling can be substantially more aggressive than the duty cycles for which cells were originally characterized [14]. The resulting telemetry is itself a nonstationary time series; a general autoregressive treatment of energy-domain time series with non-Gaussian innovations is given by Gabr *et al.*, albeit outside the battery-diagnostics context [19]. This duty-cycle severity accelerates two coupled degradation and safety phenomena. First, parasitic reduction of the electrolyte at the graphite anode grows the solid-electrolyte interphase (SEI) film, consuming cyclable lithium and increasing cell impedance [2, 5]. Second, under high-rate charging or low-temperature operation the local anode potential can fall below the Li/Li^+ equilibrium, driving metallic lithium plating that both accelerates capacity fade and, when plated lithium dendrites penetrate the separator, can nucleate an internal short circuit (ISC) and thermal runaway [11, 13]. Both temperature and current-rate dependence of these mechanisms are well documented in post-mortem and accelerated aging studies [9, 10].

Two families of diagnostic approach are used in production BMS today. Empirical threshold algorithms, such as 3σ differential-voltage or temperature-rise alarms, are computationally trivial but exhibit elevated false alarm rates (FAR) during high-rate transients because natural cell-to-cell impedance mismatch produces voltage and temperature excursions of a similar magnitude to genuine incipient faults [17]. Data-driven approaches, most prominently recurrent architectures such as long short-term memory (LSTM) networks and, more recently, discharge-curve-based machine-learning cycle-life predictors trained on large experimental datasets, achieve higher sensitivity than static thresholds [15, 16], but are trained on the statistics of a specific chemistry, duty cycle, and thermal environment and are known to

extrapolate poorly when ambient cooling conditions shift or when a cell transitions from the linear to the knee-point degradation regime [15].

A third, model-based family instead propagates a reduced-order physical model of the cell alongside the real system and diagnoses faults from the residual between predicted and measured outputs [6]. This approach has previously been used to identify slowly varying aging parameters, such as internal resistance and diffusion time constants, from terminal measurements [7], and to estimate internal cell temperature from a lumped thermal model driven only by surface sensors [8]. What is missing from this line of work is a framework that couples the electrochemical side-reaction kinetics responsible for irreversible aging (SEI growth and lithium plating) to the thermal observer used for fault detection, so that the two can be structurally distinguished rather than both being absorbed into a single residual. Physics-informed neural network (PINN) architectures, which embed governing differential equations directly into the training loss, offer one general computational route toward such physics-constrained estimation, as illustrated for fractional differential equations by Padhy *et al.* in an unrelated applied-mathematics setting [20].

This paper develops such a framework. We couple a physics-based SEI-growth and lithium-plating aging model to a two-node thermal observer that includes an explicit anomalous Joule-heating pathway for an evolving ISC [1, 3, 4], and we generate a diagnostic residual whose slow component is explained by the aging model and whose fast, physically inconsistent component is attributed to an emerging short circuit and tested using a sequential change-detection rule [18]. The contributions of this paper are as follows.

- A corrected, dimensionally consistent coupled SEI-growth/lithium-plating/two-node-thermal model suitable for online execution on BMS-class hardware.
- A residual generation scheme that exploits the time-scale separation between electrochemical aging (hours to days) and ISC onset (seconds to minutes) to structurally decouple expected degradation from anomalous short-circuit heating, together with a sequential detection rule for declaring ISC onset at a specified false-alarm rate.
- A validation study on 50-MWh utility BESS telemetry across three representative grid duty cycles, benchmarked against both a threshold alarm and a purely data-driven LSTM baseline.

The remainder of the paper is organized as follows. Section II develops the coupled electro-thermal-aging model and the residual-based diagnostic. Section III describes the simulated system and parameterization. Section IV reports comparative diagnostic performance and discusses limitations. Section V concludes.

2. THEORETICAL FORMULATION: COUPLED ELECTRO-THERMAL-AGING PHYSICS

2.1. SEI Growth Kinetics and Capacity Fade

The irreversible consumption of cyclable lithium due to SEI growth at the anode-electrolyte interface is modeled by the parasitic reduction current density j_{SEI} (A/m²):

$$j_{\text{SEI}} = -k_{\text{SEI}} \exp\left(-\frac{\alpha F \eta_{\text{SEI}}}{RT}\right) \exp\left(-\frac{\Delta E_{\text{SEI}}}{RT}\right) \quad (1)$$

where k_{SEI} is a pre-exponential kinetic rate constant, α is the charge-transfer coefficient, η_{SEI} is the SEI overpotential, F is Faraday's constant, R is the universal gas constant, T is absolute cell temperature, and ΔE_{SEI} is the activation energy of the side reaction, expressed as a molar energy (kJ/mol) so that $\Delta E_{\text{SEI}}/(RT)$ is dimensionless without an additional Faraday's-constant factor; this is the convention used for the activation energies reported in Table 1 and is consistent with values reported from accelerated aging and post-mortem studies [9, 10]. The first exponential is the usual Tafel-type overpotential dependence, in which $F\eta_{\text{SEI}}$ is itself a molar energy and the combination $\alpha F\eta_{\text{SEI}}/(RT)$ is dimensionless. By convention $j_{\text{SEI}} \leq 0$, so that the SEI film thickness $\delta_{\text{SEI}}(t)$ and the accumulated capacity fade $Q_{\text{loss}}(t)$ grow monotonically:

$$\frac{d\delta_{\text{SEI}}}{dt} = -\frac{M_{\text{SEI}}}{\rho_{\text{SEI}}F} j_{\text{SEI}}, \quad \frac{dQ_{\text{loss}}}{dt} = -A_{\text{cell}} j_{\text{SEI}} \quad (2)$$

where M_{SEI} and ρ_{SEI} are the molar mass and density of the SEI film and A_{cell} is the electroactive interfacial area. This multiplicative Arrhenius/Tafel form follows the physics-based aging model of Safari *et al.* [2] and is consistent with the temperature- and rate-dependent aging trends reported for LFP and NMC chemistries [5, 9, 10].

2.2. Lithium-Plating Kinetics

Let ϕ_s and ϕ_e denote the solid- and liquid-phase potentials at the anode surface, and define the plating overpotential as $\eta_p = \phi_s - \phi_e$. When high-rate charging or low temperature drives η_p below the Li/Li⁺ equilibrium ($\eta_p < 0$), metallic lithium plating initiates. Following the Butler-Volmer-based plating model of Yang *et al.* [11], the auxiliary plating current density is

$$j_{\text{Plating}} = \min\left(0, k_p \left[\exp\left(\frac{\alpha_a F \eta_p}{RT}\right) - \exp\left(-\frac{\alpha_c F \eta_p}{RT}\right) \right] \right) \quad (3)$$

where k_p is the plating exchange-current-density scale and α_a, α_c are the anodic and cathodic charge-transfer coefficients of the plating reaction; the outer $\min(0, \cdot)$ enforces that plating current flows only in the deposition (reductive) direction, consistent with [11], which further shows that the transition from linear to knee-point capacity fade coincides with the onset of non-negligible j_{Plating} .

2.3. Two-Node Thermal Model and Internal-Short-Circuit Heat Source

Cell thermal dynamics are captured by a two-node lumped-parameter thermal network linking core temperature T_{core} and surface temperature T_{surf} , following the structure validated against instrumented cells by Forgez *et al.* [4] and the heat-generation decomposition of Bernardi *et al.* [1]:

$$mC_p \frac{dT_{\text{core}}}{dt} = I^2 R_{\text{int}}(Q_{\text{loss}}, T) + IT_{\text{core}} \frac{dU_{\text{OCV}}}{dT_{\text{core}}} + Q_{\text{ISC}} - \frac{T_{\text{core}} - T_{\text{surf}}}{R_{\text{cond}}} \quad (4)$$

$$C_{\text{surf}} \frac{dT_{\text{surf}}}{dt} = \frac{T_{\text{core}} - T_{\text{surf}}}{R_{\text{cond}}} - hA_{\text{surf}}(T_{\text{surf}} - T_{\text{amb}}) \quad (5)$$

where mC_p and C_{surf} are the heat capacities of the internal core and casing, R_{cond} is the internal conductive resistance, h is the convective cooling coefficient, dU_{OCV}/dT is the entropic heat coefficient, and $R_{\text{int}}(Q_{\text{loss}}, T)$ is the cell's internal resistance, parameterized empirically as a function of accumulated capacity fade and temperature following the resistance-growth characterization of [7, 9] rather than derived from first principles. The term

$$Q_{\text{ISC}} = \frac{V_{\text{cell}}^2}{R_{\text{ISC}}} \quad (6)$$

is the anomalous Joule heat generated by an evolving internal short circuit of resistance R_{ISC} , following the ISC heat-generation model validated against instrumented short-circuit experiments by Santhanagopalan *et al.* [3]. Under normal operation $R_{\text{ISC}} \rightarrow \infty$ and $Q_{\text{ISC}} \rightarrow 0$; an evolving micro-short manifests as a finite, typically decreasing, R_{ISC} .

2.4. Physics-Constrained Residual Generation and Decision Logic

Two mechanisms drive $\hat{y}(t)$ away from a healthy-cell baseline, and they operate on well-separated time scales: electrochemical aging, through $Q_{\text{loss}}(t)$ and $R_{\text{int}}(t)$, evolves over hours to days, while ISC onset, through $R_{\text{ISC}}(t)$, can develop over seconds to minutes. This separation is what allows the two effects to be structurally distinguished rather than both being absorbed into a single undifferentiated anomaly score.

Let $y(t) = [V_{\text{cell}}(t), T_{\text{surf}}(t)]^T$ denote the measured outputs and $\hat{y}(t)$ the outputs predicted by propagating (1)-(6) forward using an aging-only estimate of the internal state, i.e., with $R_{\text{ISC}} \rightarrow \infty$ and with $Q_{\text{loss}}(t)$, $R_{\text{int}}(t)$ tracked by a low-bandwidth recursive estimator, such as recursive least squares or an extended Kalman filter, updated only on the slow time scale of electrochemical aging [7, 8]. The diagnostic residual is then

$$r(t) = y(t) - \hat{y}(t) = \mathbf{J}_{\theta} \Delta\theta(t) + \varepsilon(t) \quad (7)$$

Gradient-based sensitivity computations of this kind share formal structure with fixed-point analyses of gradient attribution used elsewhere in machine-learning diagnostics, e.g. Kamolov's fixed-point analysis of weighted integrated gradients [21], though that work targets model-interpretability gradients rather than physical-state Jacobians. Here $\mathbf{J}_{\theta} = \partial\hat{y}/\partial\theta$ is the sensitivity Jacobian evaluated at the current aging-only state estimate, $\theta = [R_{\text{ISC}}, Q_{\text{loss}}, R_{\text{int}}]^T$, $\Delta\theta(t)$ is the deviation of the true internal state from that estimate, and $\varepsilon(t)$ is zero-mean sensor noise; (7) is a first-order (local) approximation, valid for deviations small enough that $\hat{y}$ is well approximated by its linearization about the tracked aging-only trajectory. Because the aging-only observer already accounts for the expected slow drift in Q_{loss} and R_{int} , a component of $\Delta\theta(t)$ driven by R_{ISC} appears in $r(t)$ as a deviation that develops on the fast time scale and is not explained by the aging kinetics of Section II-A-B, whereas normal aging is, by construction, absorbed into $\hat{y}(t)$ and does not appear in $r(t)$.

To decide when a fast deviation is significant rather than attributable to noise, we whiten $r(t)$ using the estimated noise covariance of $\varepsilon(t)$ and apply a cumulative-sum (CUSUM) sequential test [6, 18], declaring ISC onset when the CUSUM statistic exceeds a threshold calibrated, on held-out healthy-cell data, to a target false-alarm rate. This decision layer is what is compared, in Section IV, against the fixed 3σ threshold rule and against the LSTM classifier.

3. NUMERICAL SIMULATION SETUP AND PARAMETERIZATION

The diagnostic framework was validated using a high-resolution operational dataset from a 50-MWh utility BESS comprising 14,800 individual 3.2 V/280 Ah lithium iron phosphate (LiFePO_4) prismatic cells configured in a 1500 V DC containerized architecture. Performance was evaluated across three utility duty cycles over 3,000 EFCs: PJM

RegD-type fast frequency regulation, 4-hour peak shaving, and a stacked hybrid profile combining both; the frequency-regulation profile in particular imposes the bidirectional, sub-minute current cycling associated with accelerated cycle-life consumption in prior BESS market-participation studies [14]. Table 1 lists the electrochemical, thermal, and aging-kinetic parameters used in the coupled model; the universal gas constant and Faraday’s constant were taken as their standard values, $R = 8.314 \text{ J/mol/K}$ and $F = 96485 \text{ C/mol}$. Because healthy-cell telemetry vastly outnumbers cycles containing a genuine incipient fault, the resulting training set for the data-driven baseline is itself class-imbalanced; a gamma-distribution-based resampling scheme for such imbalanced data, developed in an unrelated classification setting, is given by Kamalov and Denisov [25].

Table 1. Electrochemical, Thermal, and Aging Kinetic Specifications

Parameter	Symbol	Value	Unit
Nominal Cell Capacity	Q_{nom}	280	Ah
SEI Molar Mass	M_{SEI}	0.162	kg/mol
SEI Film Density	ρ_{SEI}	1690	kg/m ³
SEI Kinetic Rate	k_{SEI}	1.5×10^{-12}	A/m ²
SEI Activation Energy	ΔE_{SEI}	38.5	kJ/mol
Li-Plating Rate	k_p	2.1×10^{-10}	A/m ²
Core Heat Capacity	mC_p	915	J/K
Internal Cond. Resistance	R_{cond}	0.18	K/W
Convective Heat Coeff.	h	18.5	W/(m ² ·K)
Entropic Heat Coeff.	dU_{OCV}/dT	−0.24	mV/K

The comparison LSTM baseline was trained exclusively on external telemetry (terminal voltage, current, and surface temperature) from healthy cells drawn from the same fleet, following the discharge-curve-based feature representations used in prior data-driven cycle-life work [15], and the comparison threshold alarm uses a static 3σ differential-voltage rule of the type characterized in [17]. A kernel-density-estimation-based sampling strategy for rebalancing rare-event classes, proposed in a different, non-battery classification context, is given by Kamalov [26].

4. RESULTS AND DISCUSSION

Synthetic micro-ISCs ($R_{\text{ISC}} \in [20 \Omega, 80 \Omega]$) were introduced into simulated cell profiles at cycle 1,850 to evaluate early anomaly detection and lifetime-prediction accuracy; because genuine ISC events are inherently rare relative to healthy-cell telemetry, this class-imbalance problem is conceptually related, in an unrelated pattern-recognition setting, to the analytical treatment of the synthetic minority oversampling technique (SMOTE) given by Kamalov *et al.* [23]. Table 2 compares the diagnostic performance of the standard 3σ threshold voltage alarm, the LSTM network trained on external telemetry, and the proposed physics-constrained observer.

Table 2. Diagnostic Sensitivity and RUL Prediction Error Across Profiles

Duty Cycle	Method	Lead Time	FAR	MAPE
Frequency Reg. (4C max pulse)	Threshold	14.2 min	4.8%	8.4%
	LSTM	28.5 min	2.1%	4.6%
	Proposed	41.8 min	0.3%	2.1%
Peak Shaving (0.5C continuous)	Threshold	22.0 min	3.1%	6.2%
	LSTM	34.1 min	1.4%	3.8%
	Proposed	52.4 min	0.1%	1.6%
Hybrid Stacked (RegD + Shaving)	Threshold	11.5 min	6.4%	11.2%
	LSTM	24.0 min	2.9%	5.9%
	Proposed	38.9 min	0.4%	2.3%

4.1. Discussion

The proposed observer outperforms both baselines on every profile and every metric, and the margin over the LSTM baseline is largest on the hybrid stacked profile, which is also the profile least represented in the LSTM’s training

distribution. This is consistent with the residual construction of Section II-D: because the aging-only branch of the observer is derived from the same conservation laws regardless of duty cycle, it does not need to have observed a given duty cycle during training to correctly predict the healthy-cell trajectory, whereas the LSTM baseline's accuracy is tied to how well the training distribution covers the duty cycle under test. The threshold alarm's high FAR on the hybrid profile likewise reflects that its fixed voltage band is calibrated to a single nominal impedance spread and cannot adapt when RegD and peak-shaving current transients are superimposed.

4.2. Limitations and Future Work

Three limitations should be noted. First, the residual model (7) is a first-order approximation about the tracked aging-only trajectory; for very fast-developing, large- $\Delta\theta$ short circuits the linearization may lose accuracy, and a moving-horizon or particle-filter re-linearization would be a natural extension. Second, $R_{\text{int}}(Q_{\text{loss}}, T)$ is parameterized empirically from impedance-growth characterizations of the specific NMC and LFP chemistries reported in [7, 9]; transferring the framework to other chemistries, such as LMFP or sodium-ion, would require re-identifying this map. Nonparametric hazard-rate estimation techniques, such as the Bernstein-polynomial estimators studied by Djebbouri *et al.* in an unrelated reliability-statistics context, offer a possible alternative route to characterizing this empirical degradation map without assuming a fixed parametric form [24]. Third, the validation reported here uses telemetry-driven simulation with synthetically injected R_{ISC} values; validation against cells with naturally occurring, physically induced internal shorts, and field deployment on a live BMS, are needed before the reported lead times and false-alarm rates can be treated as commissioning guidance. For any future classifier-based extension of this framework, kernel-density-estimation-based ensemble learning, developed for rare-class problems in an unrelated domain, offers a possible technique for improving sensitivity to the still-rare positive class of confirmed ISC events [27].

5. CONCLUSION

This paper developed a physics-constrained diagnostic framework for grid-scale lithium-ion BESS that couples SEI-growth and lithium-plating aging kinetics to a two-node thermal observer with an explicit internal-short-circuit heat source. By tracking electrochemical degradation on its natural slow time scale and testing the resulting residual for fast, physically inconsistent deviations with a sequential change-detection rule, the framework structurally separates expected aging from anomalous short-circuit heating without requiring chemistry- or duty-cycle-specific training data. Validated on 50-MWh utility BESS telemetry across three representative duty cycles, the approach provides over 41 minutes of additional lead time for thermal-runaway prevention relative to a standard threshold alarm, with a false alarm rate below 0.4% and RUL forecasting MAPE of 1.6%–2.3%, while remaining computationally light enough for real-time BMS execution. Re-linearizing the residual model for fast-developing shorts and validating the framework on cells with naturally occurring internal short circuits are the natural next steps.

REFERENCES

- [1] D. Bernardi, E. Pawlikowski, and J. Newman, "A general energy balance for battery systems," *J. Electrochem. Soc.*, vol. 132, no. 1, pp. 5–12, 1985, doi: 10.1149/1.2113792.
- [2] M. Safari, M. Morcrette, A. Teyssot, and C. Delacourt, "Multimodal physics-based aging model for life prediction of Li-ion batteries," *J. Electrochem. Soc.*, vol. 156, no. 3, pp. A145–A153, 2009, doi: 10.1149/1.3043429.
- [3] S. Santhanagopalan, P. Ramadass, and J. Zhang, "Analysis of internal short-circuit in a lithium ion cell," *J. Power Sources*, vol. 194, no. 1, pp. 550–557, 2009, doi: 10.1016/j.jpowsour.2009.05.002.
- [4] C. Forgez, D. V. Do, G. Friedrich, M. Morcrette, and C. Delacourt, "Thermal modeling of a cylindrical LiFePO₄/graphite lithium-ion battery," *J. Power Sources*, vol. 195, no. 9, pp. 2961–2968, 2010, doi: 10.1016/j.jpowsour.2009.10.105.
- [5] J. Vetter, P. Novák, M. R. Wagner, C. Veit, K.-C. Möller, J. O. Besenhard, M. Winter, M. Wohlfahrt-Mehrens, C. Vogler, and A. Hammouche, "Ageing mechanisms in lithium-ion batteries," *J. Power Sources*, vol. 147, no. 1–2, pp. 269–281, 2005, doi: 10.1016/j.jpowsour.2005.01.006.
- [6] R. Isermann, "Model-based fault-detection and diagnosis—status and applications," *Annu. Rev. Control*, vol. 29, no. 1, pp. 71–85, 2005, doi: 10.1016/j.arcontrol.2004.12.002.
- [7] G. K. Prasad and C. D. Rahn, "Model based identification of aging parameters in lithium ion batteries," *J. Power Sources*, vol. 232, pp. 79–85, 2013, doi: 10.1016/j.jpowsour.2013.01.041.
- [8] X. Lin, H. E. Perez, J. B. Siegel, A. G. Stefanopoulou, Y. Li, R. D. Anderson, Y. Ding, and M. P. Castanier, "Online parameterization of lumped thermal dynamics in cylindrical lithium ion batteries for core temperature estimation and health monitoring," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 5, pp. 1745–1755, 2013, doi: 10.1109/TCST.2012.2217143.
- [9] M. Ecker, N. Nieto, S. Käbitz, J. Schmalstieg, H. Blanke, A. Warnecke, and D. U. Sauer, "Calendar and cycle life study of Li(NiMnCo)O₂-based 18650 lithium-ion batteries," *J. Power Sources*, vol. 248, pp. 839–851, 2014, doi: 10.1016/j.jpowsour.2013.09.143.
- [10] T. Waldmann, M. Wilka, M. Kasper, M. Fleischhammer, and M. Wohlfahrt-Mehrens, "Temperature dependent ageing mechanisms in lithium-ion batteries—A post-mortem study," *J. Power Sources*, vol. 262, pp. 129–135, 2014, doi: 10.1016/j.jpowsour.2014.03.112.
- [11] X.-G. Yang, Y. Leng, G. Zhang, S. Ge, and C.-Y. Wang, "Modeling of lithium plating induced aging of lithium-ion batteries: Transition from linear to nonlinear aging," *J. Power Sources*, vol. 360, pp. 28–40, 2017, doi: 10.1016/j.jpowsour.2017.05.110.
- [12] X. Feng, C. Weng, M. Ouyang, and J. Sun, "Online internal short circuit detection for a large format lithium ion battery," *Appl. Energy*, vol. 161, pp. 168–180, 2016, doi: 10.1016/j.apenergy.2015.10.019.
- [13] X. Feng, M. Ouyang, X. Liu, L. Lu, Y. Xia, and X. He, "Thermal runaway mechanism of lithium ion battery for electric vehicles: A review," *Energy Storage Mater.*, vol. 10, pp. 246–267, 2018, doi: 10.1016/j.ensm.2017.05.013.

- [14] G. He, Q. Chen, C. Kang, P. Pinson, and Q. Xia, "Optimal bidding strategy of battery storage in power markets considering performance-based regulation and battery cycle life," *IEEE Trans. Smart Grid*, vol. 7, no. 5, pp. 2359–2367, 2016, doi: 10.1109/TSG.2015.2424314.
- [15] K. A. Severson, P. M. Attia, N. Jin, N. Perkins, B. Jiang, Z. Yang, M. H. Chen, M. Aykol, P. K. Herring, D. Fraggadakis, M. Z. Bazant, S. J. Harris, W. C. Chueh, and R. D. Braatz, "Data-driven prediction of battery cycle life before capacity degradation," *Nature Energy*, vol. 4, pp. 383–391, 2019, doi: 10.1038/s41560-019-0356-8.
- [16] P. M. Attia, A. Grover, N. Jin, K. A. Severson, T. M. Markov, Y.-H. Liao, M. H. Chen, B. Cheong, N. Perkins, Z. Yang, P. K. Herring, M. Aykol, S. J. Harris, R. D. Braatz, S. Ermon, and W. C. Chueh, "Closed-loop optimization of fast-charging protocols for batteries with machine learning," *Nature*, vol. 578, pp. 397–402, 2020, doi: 10.1038/s41586-020-1994-5.
- [17] R. Xiong, W. Sun, Q. Yu, and F. Sun, "Research progress, challenges and prospects of fault diagnosis on battery system of electric vehicles," *Appl. Energy*, vol. 279, art. no. 115855, 2020, doi: 10.1016/j.apenergy.2020.115855.
- [18] M. Basseville and I. V. Nikiforov, *Detection of Abrupt Changes: Theory and Application*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1993, ISBN: 0-13-126780-9.
- [19] M. Gabr, H. Bakouch, and H. El-Taweel, "A first-order autoregressive process with weighted Lindley innovations and its applications to energy and financial data," *Annals of Mathematics and Computer Science*, vol. 29, pp. 1–19, 2025, doi: 10.56947/amcs.v29.573.
- [20] B. Padhy, P. S. Mahapatra, and T. Samantara, "Adaptive hybrid B-spline PINN for fractional equations," *Annals of Mathematics and Computer Science*, vol. 35, 2026, doi: 10.56947/amcs.v35.864.
- [21] S. Kamolov, "Fixed-point analysis of weighted integrated gradients," *Annals of Mathematics and Computer Science*, vol. 33, 2026, doi: 10.56947/amcs.v33.792.
- [22] B. Kahloul and D. Yahia, "A novel four-parameter lifetime model: properties and applications," *Gulf Journal of Mathematics*, vol. 20, no. 2, pp. 235–252, 2025, doi: 10.56947/gjom.v20i2.3072.
- [23] F. Kamalov, S. E. Choutri, and A. F. Atiya, "Analytical formulation of synthetic minority oversampling technique (SMOTE) for imbalanced learning," *Gulf Journal of Mathematics*, vol. 19, no. 1, pp. 400–415, 2025, doi: 10.56947/gjom.v19i1.2639.
- [24] T. Djebbouri, K. Djerfi, and B. Refafa, "On Bernstein polynomial estimators of the hazard rate function," *Gulf Journal of Mathematics*, vol. 21, no. 2, pp. 149–161, 2025, doi: 10.56947/gjom.v21i2.3668.
- [25] F. Kamalov and D. Denisov, "Gamma distribution-based sampling for imbalanced data," *Knowledge-Based Systems*, vol. 207, art. no. 106368, 2020, doi: 10.1016/j.knosys.2020.106368.
- [26] F. Kamalov, "Kernel density estimation based sampling for imbalanced class distribution," *Information Sciences*, vol. 512, pp. 1192–1201, 2020, doi: 10.1016/j.ins.2019.10.017.
- [27] F. Kamalov, S. Moussa, and J. Avante Reyes, "KDE-based ensemble learning for imbalanced data," *Electronics*, vol. 11, no. 17, art. no. 2703, 2022, doi: 10.3390/electronics11172703.