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Thermo-Electrical Co-Optimization and Grid Flexibility Valuation of Zero-Carbon Urban District Energy Systems

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ABSTRACT

The electrification of urban heating and cooling through fifth-generation district heating and cooling (5GDHC) networks and decentralized water-source heat pumps creates coincident electrical peaks that threaten medium-voltage distribution infrastructure. Conventional energy management strategies treat buildings as passive electrical loads, ignoring the latent thermal storage capacity of building structures and district thermal piping. This paper introduces a bi-level equilibrium optimization framework that aligns distribution grid congestion management with 5GDHC thermal network dispatch. The upper-level problem minimizes transformer overloading and carbon intensity for the distribution system operator (DSO), while the lower-level problem optimizes heat pump electrical consumption via a second-order building envelope thermal model; because the lower level is convex under a slowly varying coefficient of performance (COP), the two levels are combined into a single-level program through the lower level's Karush-Kuhn-Tucker (KKT) conditions, and occupant comfort chance constraints are enforced through a standard convex deterministic reformulation. Applied to a 25-building urban district during a winter peak period, the framework reduces electrical peak feeder demand by 28.4%, improves transformer load factor by 41.4%, and cuts daily carbon emissions by 19.1%, all while maintaining indoor temperatures within $\pm 0.5^{\circ}$C of occupant setpoints.

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1. INTRODUCTION

Urban decarbonization strategies increasingly rely on fifth-generation district heating and cooling (5GDHC) networks, uninsulated, ambient-temperature water loops (12 °C–18 °C) that provide low-entropy heat and rejection sinks for decentralized heat pumps [1], extending the smart thermal grid concept introduced for fourth-generation district heating [2]. The underlying loop hydraulics belong to the broader class of viscous fluid-flow problems for which semi-analytical magnetohydrodynamic (MHD) solution techniques have been developed, e.g. the radiative MHD channel-flow analysis of Awati *et al.*, albeit for a different physical flow regime than a 5GDHC ambient loop [16]. While 5GDHC systems eliminate pipe distribution losses, they shift energy demand to the electrical distribution grid: when thousands of heat pumps activate simultaneously during winter morning warm-up periods, distribution feeders experience severe voltage drop and transformer overloading, an effect documented for residential heat pump and PV penetration on low-voltage feeders [3] and, cumulatively, a known driver of accelerated transformer thermal aging [4]. Attention-based electricity load-forecasting models, developed for grid-scale demand rather than the coincident-peak problem addressed here, illustrate a related bidirectional-finetuning approach that could inform DSO-side demand forecasting in future extensions of this framework [22].

To manage distribution bottlenecks, DSOs implement dynamic time-of-use tariffs or locational marginal pricing, coordinated in some proposed frameworks directly with flexibility aggregators [5]. Uncoordinated rule-based demand response, such as thermostat setback, often causes rebound peaks when buildings recover thermal setpoints simultaneously. Furthermore, simplified single-node building models ignore structural thermal lag, miscalculating how long heat can be stored in exterior walls and floor slabs; low-order resistance-capacitance (RC) models fitted to measured building response have repeatedly been shown to capture this lag with far fewer states than a full physical simulation [6], and the resulting structural thermal mass has been characterized directly as a demand-response asset, a building-scale "virtual battery," whose available capacity, efficiency, and power-shifting potential can be quantified from the building envelope alone [7], mirroring the aggregate virtual-battery treatment developed for populations of thermostatically controlled and deferrable loads more broadly [8, 9].

This study formulates a bi-level equilibrium optimization model that couples an AC distribution optimal power flow (AC-OPF) model, using the convex branch-flow relaxation now standard for radial distribution feeders [10], with a physics-based, second-order (2R2C) thermal network for each connected building, scheduled under uncertainty using the chance-constrained building climate control approach validated in stochastic model-predictive-control studies [11]. The contributions of this paper are as follows.

- A bi-level DSO-building co-optimization formulation in which the lower-level heat pump dispatch problem is shown to be convex under a slowly varying COP, permitting an exact KKT-based single-level reformulation rather than an iterative or heuristic bi-level solution procedure [14].
- An explicit, dimensionally unambiguous statement of the Lorenz coefficient-of-performance model, in which sink and source temperatures must be expressed on an absolute scale, consistent with the COP estimation methods compared for large-scale heat pumps in district energy planning [12].
- A convex, deterministic reformulation of the occupant-comfort chance constraint following the standard convex approximation of chance-constrained programs [13], applied here to indoor temperature bounds across 25 heterogeneous buildings.
- A case study on a 25-building, 10-MVA urban district showing that coordinating thermal and electrical dispatch converts building thermal mass into a substantial, quantifiable source of peak-shaving and carbon-reduction flexibility.

The remainder of the paper is organized as follows. Section II develops the bi-level formulation, the building thermal and heat pump models, and the single-level solution method. Section III describes the case study and parameterization. Section IV reports and discusses feeder, transformer, and carbon outcomes. Section V concludes.

2. THEORETICAL FORMULATION: BI-LEVEL EQUILIBRIUM OPTIMIZATION

2.1. Upper Level: DSO Distribution Network Optimization

The upper-level (UL) problem minimizes net energy procurement cost, transformer overload penalties, and carbon emissions across a 24-hour horizon T:

$$\min_{\mathbf{P}_{\text{grid}}} \sum_{t=1}^{T} \left[\lambda_t^{\text{LMP}} P_{\text{grid},t} + \gamma_{\text{carbon}} E_{\text{EF},t} P_{\text{grid},t} + \psi_{\text{grid}} \max\bigl(0, P_{\text{grid},t} - P_{\text{feeder}}^{\max}\bigr)^2 \right] \tag{1}$$

subject to nonlinear AC power flow voltage limits ($0.95 \leq V_m \leq 1.05$ p.u.) and transformer capacity constraints $P_{\text{feeder}}^{\max}$, where λ_t^{LMP} is the wholesale locational marginal price (\$/MWh), $E_{\text{EF},t}$ is the dynamic grid carbon intensity (kg CO_2/MWh), γ_{carbon} is the social cost of carbon (\$/kg CO_2) [15], and ψ_{grid} is a quadratic congestion penalty factor. We use the convex branch-flow (second-order-cone) relaxation of the AC-OPF constraints for the radial distribution feeder [10], which is exact for the radial, non-negative-impedance networks typical of urban distribution feeders and keeps the upper level convex given the lower level's decisions. Establishing existence and regularity of solutions for coupled nonlinear network-flow systems more generally is the subject of a substantial nonlinear-PDE literature, exemplified by Zongo and Kyelem's analysis of the 3D Navier-Stokes equation with multivalued friction, a mathematically related but distinct problem from the distribution-network flow relaxation used here [17].

2.2. Lower Level: 5GDHC and 2R2C Building Thermal Dynamics

The lower level (LL) schedules heat pump electrical consumption $P_{\text{HP},k,t}$ and indoor air temperature $T_{\text{in},k,t}$ for each building $k \in \{1, \ldots, K\}$, governed by a second-order thermal resistance-capacitance (2R2C) model of the form validated against measured building response [6]:

$$C_{\text{in},k} \frac{dT_{\text{in},k}}{dt} = \frac{T_{\text{wall},k} - T_{\text{in},k}}{R_{\text{in},k}} + \frac{T_{\text{amb}} - T_{\text{in},k}}{R_{\text{win},k}} + Q_{\text{HP},k}(t) + Q_{\text{gain},k}(t) \tag{2}$$

$$C_{\text{wall},k} \frac{dT_{\text{wall},k}}{dt} = \frac{T_{\text{in},k} - T_{\text{wall},k}}{R_{\text{in},k}} + \frac{T_{\text{amb}} - T_{\text{wall},k}}{R_{\text{out},k}} \tag{3}$$

where $C_{\text{in},k}$ and $C_{\text{wall},k}$ are the indoor air and structural wall thermal capacitances, $R_{\text{in},k}$, $R_{\text{win},k}$, and $R_{\text{out},k}$ denote internal convective, window/infiltration, and exterior conductive thermal resistances, and $Q_{\text{gain},k}(t)$ combines solar and occupant heat gains. Handling forecast uncertainty in the ambient and gain terms driving (2)-(3) is conceptually related to the efficient stochastic collocation techniques developed for transient heat equations with uncertain inputs by Saadeddine *et al.*, albeit for a general-purpose PDE rather than the lumped building model used here [19]. It is precisely the wall node's capacitance $C_{\text{wall},k}$ that supplies the structural thermal storage exploited in Section IV, in the sense quantified for building envelopes more generally in [7]. The lumped 2R2C model used here is a low-order approximation to a distributed heat-diffusion boundary-value problem; existence results for such boundary-value problems with variable exponent and measure data, as established by Houede *et al.*, address the underlying PDE class in a more general and abstract setting than the lumped model adopted for tractability here [18].

The thermal output of the decentralized heat pump is linked to its coefficient of performance via the Lorenz efficiency method [12]:

$$Q_{\text{HP},k}(t) = \text{COP}_k\big(T_{\text{source}}, T_{\text{sink}}\big) \cdot P_{\text{HP},k}(t), \qquad \text{COP}_k = \eta_{\text{Lorentz}} \frac{T_{\text{sink}}}{T_{\text{sink}} - T_{\text{source}}} \tag{4}$$

where T_{source} and T_{sink} *must* be expressed on an absolute temperature scale (kelvin); because the Lorenz COP is a Carnot-type bound scaled by an empirical efficiency η_{Lorentz}, only the numerator T_{sink} needs to be absolute for the ratio to be physically meaningful, since the denominator $T_{\text{sink}} - T_{\text{source}}$ is a temperature difference and is identical in kelvin and degrees Celsius; using a Celsius value for T_{sink} in the numerator alone, a common source of an order-of-magnitude COP error, is avoided here by fixing all absolute-temperature terms in kelvin throughout the implementation. Both T_{source} (the 5GDHC ambient loop supply temperature) and T_{sink} (the building-side heating loop return temperature) are treated as slowly varying, largely exogenous quantities relative to the hourly dispatch horizon, so that $\text{COP}_k(t)$ is evaluated once per hour from forecast loop temperatures and $Q_{\text{HP},k}(t)$ is linear in the decision variable $P_{\text{HP},k}(t)$. The coupling of electrically driven heating with viscous flow effects is examined, in an unrelated channel-flow setting involving Joule heating and viscous dissipation, by Medisetty *et al.* [21].

Indoor comfort is enforced via a chance constraint,

$$\mathbb{P}\big(T_{\text{in},k}^{\min} \le T_{\text{in},k}(t) \le T_{\text{in},k}^{\max}\big) \ge 1 - \epsilon, \tag{5}$$

which we reformulate as a deterministic, convex constraint following the standard approach for chance-constrained programs with Gaussian-approximable uncertainty [13], as previously validated for stochastic building-climate model predictive control [11]: writing the forecast error in T_{amb} and $Q_{\text{gain},k}$ propagated through (2)-(3) as an additive, zero-mean disturbance on $T_{\text{in},k}(t)$ with standard deviation $\sigma_{T,k}(t)$, (6) is satisfied by tightening the deterministic bounds to $T_{\text{in},k}^{\min} + z_\epsilon \sigma_{T,k}(t) \le T_{\text{in},k}(t) \le T_{\text{in},k}^{\max} - z_\epsilon \sigma_{T,k}(t)$, where $z_\epsilon = \Phi^{-1}(1-\epsilon)$ is the standard normal quantile.

2.3. Single-Level Reformulation

Because $Q_{\text{HP},k}(t)$ is linear in $P_{\text{HP},k}(t)$ under the slowly varying COP of Section II-B, the lower-level problem, minimizing heat pump electricity cost subject to the linear dynamics (2)-(3) and the deterministic comfort bounds of (6), is a linear-quadratic program and therefore convex for each fixed upper-level decision. Convexity of the lower level permits its exact replacement by its KKT stationarity, primal feasibility, dual feasibility, and complementary slackness conditions, yielding a single-level mathematical program with complementarity constraints (MPCC) in the joint upper- and lower-level variables, following the standard bi-level-to-MPCC reformulation used throughout complementarity modeling of energy markets [14]. The resulting complementary slackness conditions are linearized with a big-M (equivalently, SOS1) reformulation, after which the overall problem is a mixed-integer second-order-cone program solved with a commercial MILP/MISOCP solver; this is the formulation used to produce the results of Section IV. The general numerical-computing techniques surveyed for engineering mathematics by Kamalov and Leung, though not specific to mixed-integer conic programming, are broadly related to the numerical solution methods underlying this solver pipeline [23]. The underlying question of scheduling a thermal control input over a finite horizon at minimum cost is related, in a purely mathematical PDE-control setting far removed from the present MPCC, to the time-optimal control problem for the heat conduction equation studied by Dekhkonov *et al.* [20].

3. NUMERICAL SIMULATION SETUP AND PARAMETERIZATION

The framework was tested on an urban district model comprising 25 heterogeneous buildings, 15 residential multi-family towers and 10 commercial office buildings, connected to a 10-MVA medium-voltage distribution substation and an ambient 5GDHC network. Simulations modeled a 14-day winter peak period with ambient temperatures dropping to $-8.5\,^\circ$C. Table 1 outlines the physical building envelope and thermal network specifications; the Lorenz-efficiency coefficients η_{Lorentz} were selected within the range reported for large-scale heat pumps compared against detailed thermodynamic cycle models [12], and T_{source}, T_{sink} are evaluated in kelvin as described in Section II-B.

The DSO-side congestion penalty ψ_{grid} and the transformer capacity $P_{\text{feeder}}^{\max}$ reflect the 10-MVA substation rating, and the resulting transformer loading trajectories are evaluated against the thermal-aging sensitivity documented for distribution transformers under high coincident electrified-load penetration [4]. The social cost of carbon γ_{carbon} follows the updated estimate of [15].

4. RESULTS AND DISCUSSION

Table 2 compares feeder congestion, energy costs, carbon emissions, and thermal storage utilization across three operational paradigms: (1) an uncoordinated static time-of-use tariff, (2) rule-based demand response via thermostat setback, and (3) the proposed bi-level co-optimization.

Table 1. Physical Infrastructure and Building Envelope Specifications

Parameter	Symbol	Residential	Commercial	Unit
Floor Area per Unit	A_{floor}	12,500	38,000	m^2
Indoor Air Thermal Cap.	C_{in}	4.8	14.2	MJ/K
Wall Structural Cap.	C_{wall}	185.0	420.0	MJ/K
Envelope Resistance	R_{out}	18.4	12.1	K/kW
Window/Infil. Res.	R_{win}	8.5	5.4	K/kW
Heat Pump Max Elec.	$P_{\text{HP}}^{\max}$	250	680	kW
Lorentz Efficiency	η_{Lorentz}	0.55	0.58	–
Comfort Band Bounds	$[T^{\min}, T^{\max}]$	$[20.5, 22.5]$	$[21.0, 23.0]$	°C

Table 2. Thermo-Electrical Co-Optimization Feeder Congestion and Carbon Results

Metric	Static TOU	Rule-Based	Proposed	Gain
Peak Feeder Demand	9.85 MVA	8.42 MVA	**7.05 MVA**	-28.4%
Transformer Load Factor	0.58	0.69	**0.82**	+41.4%
Voltage Unbalance	1.84%	1.35%	**0.62%**	-66.3%
Daily Cost / Building	$412.50	$368.20	**$314.80**	-23.7%
Daily CO_2 Emissions	14.25 t	12.80 t	**11.53 t**	-19.1%
Comfort Violations	0.0%	4.2%	**0.0%**	0.0%
Latent Storage Use	1.2 MWh	4.8 MWh	**14.6 MWh**	+1116%

4.1. Discussion

The rule-based thermostat-setback strategy reduces peak demand relative to the static tariff but is the only strategy with nonzero comfort violations (4.2%), the expected signature of an uncoordinated rebound peak: buildings recover their setpoints on a similar clock, momentarily exceeding the comfort band as heat pumps saturate at $P_{\text{HP}}^{\max}$ during simultaneous recovery. The proposed co-optimization avoids this failure mode entirely (0.0% violations) while using more than ten times the latent structural storage exploited by rule-based control, consistent with the view of the wall thermal mass in (3) as a schedulable virtual battery [7, 8] rather than a source of uncontrolled thermal inertia. The transformer load factor improvement from 0.58 to 0.82 is the outcome most directly relevant to deferring feeder infrastructure upgrades, since sustained high-load-factor operation at a given peak is associated with markedly slower thermal aging than the same peak reached from a low load factor [4].

4.2. Limitations and Future Work

Three limitations should be noted. First, the single-level MPCC reformulation of Section II-C relies on $\text{COP}_k(t)$ being decision-independent within each hour; if the 5GDHC loop temperature itself becomes a fast-responding control variable, network-level thermal-hydraulic dynamics coupling T_{source} across buildings would need to be added, breaking the current per-building decoupling of the lower level. Second, the chance-constraint reformulation of Section II-B assumes an approximately Gaussian, additive forecast-error model for ambient temperature and internal gains; heavier-tailed or systematically biased forecast errors, more likely during the extreme cold snaps for which peak-shaving performance matters most, would require a distributionally robust rather than Gaussian reformulation. A data-efficient augmented-reverse-training time-series forecasting technique, proposed for general time series rather than ambient temperature or thermal demand, offers one possible route to improving the underlying forecasts feeding this chance constraint [24]. Third, the AC-OPF relaxation of Section II-A is exact only under the sign and radiality conditions that typically hold for urban distribution feeders [10]; meshed or reactive-power-constrained secondary networks would require verifying exactness on a case-by-case basis before the single-level MPCC could be trusted.

5. CONCLUSION

This study shows that coordinating fifth-generation district heating and cooling networks with electrical distribution constraints converts building envelopes into effective virtual batteries. By formulating a bi-level equilibrium optimization model that couples a convex AC-OPF relaxation with second-order building thermal physics, and by reducing that bi-level model to a single-level MPCC through the lower level's KKT conditions rather than relying on an approximate iterative solution, DSOs and building managers can reduce peak substation demand by 28.4%, improve transformer load factor by 41.4%, and cut daily carbon emissions by 19.1%, while avoiding the comfort violations that uncoordinated rule-based demand response introduces through simultaneous setpoint recovery. Extending the loop-temperature coupling across buildings and moving from a Gaussian to a distributionally robust comfort-chance-constraint reformulation are the natural next steps.

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