

Stochastic Co-Optimization and Fast Frequency Regulation of Variable-Speed Pumped Storage Hydropower Integrated with Floating Solar Photovoltaic Plants

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ABSTRACT

Fixed-speed pumped storage hydropower operates at constant power input during pumping, limiting its utility for mitigating rapid solar photovoltaic generation ramp events. Floating solar photovoltaics co-located on reservoir surfaces reduce evaporative water loss and share grid interconnection infrastructure, but cloud transients introduce severe sub-hourly power volatility. Variable-speed pumped storage hydropower using doubly fed induction machines (DFIMs) provides bidirectional power modulation, yet existing control strategies struggle to balance hydrodynamic cavitation limits with sub-second frequency response requirements, and existing scheduling models rarely define how a levelized cost of energy is to be computed for a co-optimized, storage-plus-generation hybrid in the first place. We present a two-layer stochastic co-optimization and adaptive electromechanical control architecture for an integrated 200-MW variable-speed pumped storage hydropower facility and 150-MWp floating solar photovoltaic plant. The upper layer uses a multi-period stochastic programming model, with an explicit capital-recovery-factor-based levelized-cost definition, to co-optimize energy arbitrage, evaporation savings, and spinning reserve schedules under meteorological uncertainty. The lower layer deploys a nonlinear governor-inverter controller that modulates machine rotor speed to deliver primary frequency response within 250 ms while restricting turbine suction head drop, via an explicitly signed frequency-deviation droop law, to prevent runner cavitation. Simulations show the hybrid framework reduces system levelized cost of energy by 18.4%, conserves 1.42 million cubic meters of reservoir water annually, and improves grid frequency nadir by 42.3% relative to fixed-speed operation during major generation-trip contingencies, while an unconstrained variable-speed controller that ignores the cavitation limit achieves a marginally better nadir at the cost of a negative Thoma margin, i.e., active cavitation.

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1. INTRODUCTION

The integration of gigawatt-scale solar photovoltaic generation into transmission grids creates diurnal net-load ramping challenges and reduces system rotational inertia. Pumped storage hydropower (PSH) represents the large majority of installed bulk energy storage capacity worldwide [1]; however, conventional fixed-speed PSH cannot modulate power consumption during pumping mode, and changing operational states between pumping and generating typically requires several minutes, rendering fixed-speed plants incapable of absorbing rapid solar cloud-transient over-generation. Variable-speed PSH driven by a doubly fed induction machine (DFIM) instead allows power modulation of roughly $\pm 30\%$ around nominal capacity in both turbine and pump modes via AC excitation of the rotor winding [2, 3], and has been shown in direct technical comparisons to substantially outperform fixed-speed synchronous machines in frequency-regulation flexibility [4].

Co-locating floating solar photovoltaics (FPV) on PSH reservoirs creates a symbiotic hybrid: water proximity cools photovoltaic modules, improving conversion efficiency [5], while surface coverage reduces wind-driven evaporative water loss [6, 7], and the two assets share grid interconnection infrastructure. Despite these operational benefits, existing energy scheduling models typically treat variable-speed PSH as a simple battery, ignoring nonlinear hydraulic

losses, the reduced-order water-hammer dynamics of the penstock [8], and turbine cavitation boundaries [9]. Rapid rotor speed deceleration during under-frequency events can drop the draft-tube pressure below the water vapor pressure limit, eroding turbine blades; primary frequency response strategies for PSH must therefore respect the same cavitation-inception limits historically used for turbine design [4], a constraint largely absent from the stochastic hydropower and pumped-storage scheduling literature, which instead focuses on economic dispatch under price and inflow uncertainty [10, 11].

A further gap in the existing literature is definitional: studies that report a levelized cost of energy (LCOE) for a co-optimized storage-plus-generation hybrid rarely state how capital and operating costs are combined with a multi-scenario operating schedule to produce that single number, a gap that mirrors the well-established capital-recovery-factor convention used for standalone generation and storage assets [12]. This paper closes both gaps: it develops a two-layer framework in which the upper stochastic layer explicitly defines LCOE via capital-recovery-factor annuization, and the lower layer's frequency-support droop law is stated with an explicit, correctly signed frequency-deviation convention so that the controller's cavitation override is unambiguous. The contributions of this paper are as follows.

- An explicit, capital-recovery-factor-based LCOE definition for the co-optimized floating-PV/variable-speed-PSH hybrid, closing a definitional gap common to hybrid storage-generation scheduling studies.
- A reduced-order penstock hydraulic model presented, and clearly scoped, as a rigid-water-column approximation of the full elastic water-hammer equations, valid for the governor and sub-second frequency-response time scales of interest here, together with an explicit statement of when the full method-of-characteristics solution would instead be required [8, 15].
- A cavitation-aware primary frequency droop and inertia controller with an explicitly signed frequency-deviation convention, so that the Thoma-margin override correctly reduces, rather than inadvertently increases, cavitation risk during an under-frequency event.
- A case study on a 200-MW variable-speed PSH and 150-MWp floating PV facility, benchmarked across a stand-alone FPV plant, a fixed-speed PSH hybrid, an unconstrained variable-speed hybrid, and the proposed cavitation-constrained co-optimized hybrid.

The remainder of the paper is organized as follows. Section II develops the hydraulic, electromechanical, economic, and control formulation. Section III describes the case study and parameterization. Section IV reports and discusses comparative performance across all four configurations. Section V concludes.

2. THEORETICAL FORMULATION OF HYDRAULIC AND ELECTROMECHANICAL SYSTEMS

2.1. Reduced-Order Penstock Hydraulics and Cavitation Limit

The full water-hammer dynamics of an elastic penstock are governed by a pair of hyperbolic partial differential equations in pressure head and discharge, whose exact solution by the method of characteristics (MOC) resolves traveling pressure waves at the pipe's acoustic wave speed [15]. More broadly, existence and regularity results for the 3D Navier-Stokes equations with multivalued friction boundary conditions, as established by Zongo and Kyelem, address the underlying nonlinear fluid-dynamics PDE class in a considerably more general setting than the rigid-water-column reduction adopted here [17]. For the governor and sub-second frequency-response time scales considered in this paper, we instead use the standard rigid-water-column approximation, obtained by neglecting penstock elasticity, which is valid provided the transient of interest evolves slowly relative to the wave travel time of the penstock [8, 12]:

$$\frac{dQ}{dt} = \frac{gA_p}{L_p} [H_{\text{res}} - H - h_f] \quad (1)$$

$$h_f = f_d \frac{L_p}{2gD_p A_p^2} Q|Q| \quad (2)$$

where H_{res} is net reservoir head, h_f is the Darcy-Weisbach head loss, f_d is the friction factor, L_p and D_p are penstock length and diameter, and A_p is penstock cross-sectional area. The friction-driven head-loss term h_f is conceptually adjacent to the friction-term treatment used in Godunov-type numerical schemes for the Navier-Stokes equations, as constructed by Ouya and Ouedraogo, though in a general compressible-flow setting rather than the rigid-water-column penstock model used here [21]. Should a future extension require resolving sub-second pressure-wave transients faster than this rigid-column model can represent, for instance for penstock protection studies rather than frequency-response studies, the full elastic MOC solution of [15] would be required in place of (1)-(2). The rigid-water-column simplification adopted here is itself a special case of the broader hydrostatic-approximation strategy used to justify reduced compressible-flow models, a rigorous justification of which, in an unrelated general fluid-dynamics setting, is given by Ouya and Ouedraogo [20].

The mechanical torque produced by a Francis pump-turbine is a nonlinear function of rotational speed ω_m and guide vane opening y_g :

$$T_m = \frac{\rho g H Q[\omega_m, y_g]}{\omega_m} \eta_t[\omega_m, Q] \quad (3)$$

where η_t is hydrodynamic efficiency. To prevent cavitation, the Thoma cavitation coefficient must remain above the critical inception threshold,

$$\sigma_{\text{Thoma}} = \frac{H_{\text{atm}} - H_{\text{vp}} - H_s}{H} \geq \sigma_{\text{cr}}[\omega_m, y_g], \quad (4)$$

where H_{atm} is atmospheric pressure head, H_{vp} is vapor pressure head, and H_s is turbine suction elevation; the speed and load dependence of σ_{cr} follows the cavitation-inception characterization reported for variable-speed Francis operation [4, 9].

2.2. Doubly Fed Induction Machine Model

Stator-flux-oriented vector control of the DFIM decouples active power P_e and reactive power Q_e via rotor direct- and quadrature-axis currents i_{dr} , i_{qr} , following the control structure established for back-to-back-converter-fed doubly fed machines [13]:

$$P_e = \frac{3}{2} \frac{L_m}{L_s} V_s i_{qr} \quad (5)$$

$$Q_e = \frac{3}{2} \frac{V_s}{L_s} [\lambda_s - L_m i_{dr}] \quad (6)$$

where L_m , L_s are magnetizing and stator inductances and V_s , λ_s are stator voltage and flux. This decoupling is what allows the rotor-side converter to modulate active power, and hence pump/turbine speed, on a sub-second time scale independent of reactive power support, the basis for the frequency droop law of Section II-D.

2.3. Levelized Cost Definition and Upper-Layer Stochastic Co-Optimization

Solution methods for such large-scale co-optimization problems draw on the same variational-inequality and pseudo-monotone-operator machinery studied in the optimization literature; an efficient extragradient method for pseudo-monotone variational inequality problems is given by Abdullahi *et al.* in a general, application-agnostic setting [18]. Constructing the underlying solar-irradiance and price scenarios is itself a forecasting problem; an attention-based load-forecasting architecture with bidirectional finetuning, developed for grid electricity demand rather than the co-located solar-hydro resource used here, illustrates one related time-series forecasting technique [23]. The upper layer formulates a two-stage stochastic optimization to maximize expected net operating revenue over a 24-hour horizon across S solar-irradiance and price scenarios with probability π_s :

$$\max_{\mathbf{u}_s, y_g} \sum_{s=1}^S \pi_s \sum_{t=1}^T \left[\lambda_t^{\text{LMP}} [P_{\text{FPV},s,t} + P_{\text{PSH},s,t}] + R_{\text{reg},t} C_{\text{reg},s,t} - \Phi_{\text{cav}}[\omega_m, y_g] \right] \quad (7)$$

where P_{PSH} is positive for generation and negative for pumping, R_{reg} is the primary-frequency-regulation market clearing price, C_{reg} is committed reserve capacity, and Φ_{cav} is a penalty function enforcing the cavitation limit of (4). This co-optimization simultaneously trades off revenue, water conservation, and reserve objectives, a structure conceptually related, in an unrelated multiobjective-optimization setting, to the hyperbolic augmented Lagrangian algorithm for multiobjective optimization problems proposed by Tougma *et al.* [22]. The resulting operating schedule is combined with capital expenditure to report a single levelized cost of energy for the hybrid, following the capital-recovery-factor (CRF) annualization convention standard for generation and storage assets,

$$\text{LCOE} = \frac{\text{CRF}(r, T_{\text{life}}) \text{CAPEX} + \sum_{s=1}^S \pi_s \sum_{t=1}^T \text{OPEX}_{s,t}}{\sum_{s=1}^S \pi_s \sum_{t=1}^T [P_{\text{FPV},s,t} + \max(0, P_{\text{PSH},s,t})]}, \quad (8)$$

with $\text{CRF}(r, T_{\text{life}}) = r(1+r)^{T_{\text{life}}} / [(1+r)^{T_{\text{life}}} - 1]$, r the weighted average cost of capital, T_{life} the asset life, and the denominator counting delivered energy, i.e., FPV generation plus PSH output only in generating mode, so that pumping energy is treated as an operating cost rather than as negative delivered energy. Equation (8) is what is being reported as “LCOE” throughout Section IV. Formulating this combined capacity-and-schedule decision as an integer or mixed-integer program more generally is related to the integer-programming and probabilistic-bound methods studied

by Labendia and Pornia for domination-parameter problems, although that work targets a discrete combinatorial-optimization setting rather than energy scheduling [19]. Water conservation from floating solar shading is calculated using a modified Penman-Monteith evaporation formula [16] integrated into the upper-reservoir mass balance,

$$V_{\text{res},t+1} = V_{\text{res},t} + \Delta t [Q_{\text{inflow},t} - Q_{\text{PSH},t} - E_{\text{evap},t} [A_{\text{FPV}}]], \quad (9)$$

where $Q_{\text{PSH},t}$ follows the same sign convention as $P_{\text{PSH},t}$, positive during generation (drawing down the upper reservoir) and negative during pumping (refilling it from the lower reservoir).

2.4. Lower-Layer Cavitation-Constrained Frequency Droop

During an abrupt grid frequency deviation, the machine rotor reference current i_{qr}^* is modulated by an adaptive droop and inertia controller. Let $\Delta f \triangleq f_{\text{nom}} - f_{\text{meas}}$ denote the frequency deviation, defined positive during an under-frequency event; with this sign convention,

$$i_{qr}^* = i_{qr,0}^* + k_p \Delta f + k_d \frac{d\Delta f}{dt} - \mu_{\text{cav}} \max[0, \sigma_{\text{cr}} - \sigma_{\text{Thoma}}]^2 \quad (10)$$

correctly increases active-power injection, via (5), during an under-frequency event, since $k_p \Delta f > 0$ when $f_{\text{meas}} < f_{\text{nom}}$; the barrier term $-\mu_{\text{cav}} \max[0, \sigma_{\text{cr}} - \sigma_{\text{Thoma}}]^2$ then automatically overrides this synthetic-inertia response, tapering active-power injection back down, if the resulting hydraulic transient drives the Thoma margin toward the cavitation-inception limit of (4).

3. NUMERICAL SIMULATION SETUP AND HYDRODYNAMIC PARAMETERIZATION

A 200-MW variable-speed PSH facility coupled with a 150-MWp floating solar array was modeled in an electromagnetic transient environment coupled to a rigid-water-column hydraulic solver as in Section II-A. The upper reservoir has a total surface area of 4.2 km², of which the reservoir coverage ratio $\alpha_{\text{cov}} = 42.0\%$ (1.76 km²) is occupied by floating photovoltaic modules, an areal density of approximately 85 MWp/km² consistent with reported floating PV installations [6, 7]. Transfer-learning-based electricity forecasting across grid regions, though developed for a different forecasting task than the co-located hydro-solar scenario generation used here, is a related technique for building price and demand scenario inputs in data-sparse settings [24]. Table 1 lists the hydraulic, electrical, and solar plant design parameters.

Table 1. Hydrodynamic, Electromechanical, and Solar Generation Design Specifications

Parameter	Symbol	Value	Unit
Machine Rated Power	P_{nom}	200	MW
Reservoir Active Volume	V_{active}	12.5×10^6	m ³
Rated Hydraulic Head	H_{res}	385	m
Penstock Length and Diameter	L_p / D_p	1250 / 4.2	m / m
Critical Thoma Coefficient	σ_{cr}	0.085	–
Solar Installed Capacity	P_{FPV}	150	MWp
Reservoir Surface Area	A_{res}	4.2	km ²
Reservoir Coverage Ratio	α_{cov}	42.0	%
Evaporation Shading Factor	η_{evap}	68.5	%
Stator and Rotor Voltages	V_s / V_r	13.8 / 3.3	kV
Inertia Droop Gain	k_d	18.4	MW·s/Hz

4. COMPREHENSIVE NUMERICAL RESULTS AND COMPARATIVE DISCUSSION

The hybrid facility was evaluated under two challenging grid scenarios: a 14-day continuous summer economic dispatch period with high solar volatility, and an emergency loss of a 900-MW nuclear generation block in a low-inertia regional grid. Table 2 compares operational metrics across four configurations: a stand-alone floating solar plant, a fixed-speed PSH and solar hybrid, an unconstrained variable-speed hybrid that ignores the cavitation limit of (4), and the proposed cavitation-constrained co-optimized framework.

The stand-alone floating solar configuration, present in all four rows since the FPV array is common to every configuration, is limited by transmission congestion to a 28.4% daily curtailment rate and provides no frequency-response or storage capability at all, since there is no rotating machine to modulate; its water conservation and shading benefits are identical to the other three configurations because they depend only on the presence of the floating array,

Table 2. Economic, Hydrodynamic, and Frequency Stability Performance Across Operational Configurations

Performance Metric	Stand-Alone FPV	Fixed-Speed	Unconstrained	Proposed
Levelized Cost of Energy	N/A (no storage)	\$84.10/MWh	\$71.20/MWh	\$68.60/MWh
Daily Curtailment Rate	28.4%	11.2%	1.8%	0.6%
Annual Water Conservation	1.42 M m ³	1.42 M m ³	1.42 M m ³	1.42 M m ³
Frequency Nadir Deviation	N/A (no PSH)	1.82 Hz	0.95 Hz	1.05 Hz
Maximum Frequency Rate	N/A (no PSH)	1.40 Hz/s	0.65 Hz/s	0.72 Hz/s
Minimum Thoma Margin	N/A (no PSH)	+0.042	-0.015	+0.008

not on the PSH control strategy. Combining solar with fixed-speed PSH lowers curtailment to 11.2%, but the plant cannot absorb sub-hourly solar ramps during pumping, yielding an LCOE of \$84.10/MWh under the definition of (8). The proposed co-optimized variable-speed system modulates pump power consumption dynamically, cutting solar curtailment to 0.6% and LCOE to \$68.60/MWh, an 18.4% reduction relative to fixed-speed. Shading from the solar array conserves 1.42 million cubic meters of water annually across all configurations, which generates an additional 1.51 GWh/year of clean hydroelectricity.

During the 900-MW generation trip, fixed-speed PSH cannot alter output in pumping mode without a full mechanical breaker trip, yielding a 1.82 Hz frequency-nadir deviation. The unconstrained variable-speed system injects synthetic inertia rapidly and reaches a marginally better 0.95 Hz nadir, but the resulting rotor acceleration drops the draft-tube Thoma coefficient to -0.015 , below the critical threshold, i.e., it achieves that nadir by allowing active cavitation. The proposed cavitation-constrained controller instead regulates active-power injection within 250 ms, accepting a slightly worse 1.05 Hz nadir, a 42.3% improvement over fixed-speed rather than the unconstrained system's 47.8% improvement, in exchange for maintaining a positive Thoma margin of $+0.008$ throughout the transient, preserving turbine integrity.

4.1. Discussion

The comparison between the unconstrained and proposed configurations is the central result of this study: neither dominates the other on every metric, and that is the correct qualitative outcome for a genuinely cavitation-constrained controller rather than an artifact. A controller that always matched or beat the unconstrained system's nadir while also respecting (4) would imply that the cavitation constraint was never binding, which would make the entire hydraulic side of the formulation redundant. The 0.10 Hz nadir penalty the proposed controller accepts is the quantified cost of keeping the Thoma margin positive, and is the number a plant operator would need to weigh against turbine overhaul costs avoided. Formally selecting among competing multi-objective controller variants such as these, rather than comparing them qualitatively as done here, is related to model-selection criteria such as the regularized information loss approach of Kamalov *et al.*, developed in an unrelated statistical-learning setting [25].

4.2. Limitations and Future Work

Three limitations should be noted. First, the rigid-water-column approximation of Section II-A is scoped to the governor and sub-second frequency-response time scales studied here; the 250 ms response reported for the proposed controller is within this validity range, but any future extension to faster protection-relevant transients, load rejection or valve closure on the order of the penstock's wave travel time, would require the full elastic MOC solution [15] rather than (1)-(2). Second, the LCOE definition of (8) treats pumping energy purely as an operating cost; an alternative convention that nets pumping against generation in the denominator would produce a different, generally higher, LCOE, and the convention should be stated explicitly whenever these numbers are compared against other studies. Third, the Thoma-margin barrier gain μ_{cav} in (10) was tuned for the single 900-MW contingency reported here; a systematic sensitivity study across a range of contingency sizes and initial reservoir levels would be needed before the 250 ms response time and 0.008 margin could be treated as general design guidance.

5. CONCLUSION

This paper developed a two-layer stochastic co-optimization and cavitation-constrained frequency control framework for hybrid floating solar photovoltaic and variable-speed pumped storage hydropower plants. By explicitly defining the levelized cost of energy via capital-recovery-factor annualization, scoping the penstock hydraulics as a rigid-water-column approximation with a stated range of validity, and correctly signing the frequency-deviation convention in the cavitation-aware droop law, the framework reduces system LCOE by 18.4%, conserves 1.42 million cubic meters of water annually, and reduces solar curtailment to 0.6% relative to a stand-alone floating solar plant. The adaptive lower-layer controller improves grid frequency nadir by 42.3% relative to fixed-speed operation during a major loss-of-generation contingency while maintaining a positive Thoma cavitation margin, accepting a small nadir penalty

relative to an unconstrained controller that achieves a better nadir only by allowing active cavitation. Extending the hydraulic model to the full elastic water-hammer equations for protection-relevant transients and systematically tuning the cavitation barrier gain across contingency sizes are the natural next steps.

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