

Thermo-Economic and Carbon-Intensity Co-Optimization of Decentralized Bio-Hydrogen Systems in Commercial Buildings

Zhāng Mǐn¹, Wáng Jìng²

¹Department of Computer Engineering, Shaoxing University, Shaoxing, China

²Department of Electrical Engineering, Qiqihar University, Qiqihar, China

Article Info

Article history:

Received Jul 16, 2025

Revised Sep 4, 2025

Accepted Sep 25, 2025

Keywords:

Bio-Hydrogen, Commercial Buildings, Exergy Analysis, Carbon Intensity Co-Optimization, Decentralized Cogeneration.

ABSTRACT

Decentralized bio-hydrogen systems, integrating localized biomass gasification, hydrogen cogeneration, and HVAC absorption cooling, offer a viable pathway for decarbonizing commercial real estate. However, existing control strategies fail to co-optimize second-law thermodynamic exergy efficiency with dynamic grid carbon intensity and fluctuating carbon-tax pricing, a gap consistent with the broader finding that carbon-pricing frameworks only translate into emission reductions when paired with dispatchable technology able to respond to the price signal [15]. This paper formulates a nonlinear thermo-economic optimization model for building-integrated bio-hydrogen systems operating under regional emission-trading schemes, together with an explicit capital-recovery-factor-based definition of the Levelized Cost of Building Energy (LCOBE) tying the dispatch objective to the building's actual 20-year capital and financing profile. Evaluated across three distinct climate zones (Cold, Mixed, Tropical), the proposed Carbon-Exergy Co-Optimization (CECO) algorithm increases annual exergy efficiency by an average of 21% relative to conventional heat-led operation, and reduces LCOBE by up to 21.4% under a strict \$130/ton carbon-tax framework.

This is an open access article under the CC BY-SA license.



Corresponding Author:

Zhāng Mǐn

Department of Computer Engineering

Shaoxing University

Shaoxing, China

Email: zhang.min@usx.edu.cn

1. INTRODUCTION AND THEORETICAL FRAMEWORK

Commercial buildings account for a large and persistent share of urban energy demand and carbon emissions, and decarbonizing them requires technologies that can both supply heating and cooling efficiently and respond dynamically to time-varying grid conditions. Building-integrated fuel cell combined heat and power (CHP) has been assessed as one such pathway, with techno-economic studies of commercial food-retail deployments showing that fuel cell CHP can achieve competitive payback periods relative to combustion-engine CHP while substantially reducing local pollutant emissions [2]. Biomass gasification coupled to pressure-swing-adsorption (PSA) hydrogen purification offers a route to feed such systems with a renewable, storable fuel [4, 5], and green-hydrogen-fueled building CHP has separately been shown to reduce primary energy intake and greenhouse gas emissions relative to conventional building energy supply [3].

Evaluating such systems on a first-law (energy) basis alone obscures the fact that electricity, high-grade heat, low-grade heat, cooling, and chemical hydrogen are not thermodynamically equivalent; a second-law exergy analysis is required to compare them on a common basis [6, 7]. The chemical exergy of the solid biomass feedstock itself is conventionally estimated via the Szargut-Styrzyska correlation relating the exergy-to-heating-value ratio to the fuel's elemental composition [1], a method still in wide use for updated biomass and waste-derived fuel exergy assessments.

A second gap in the literature concerns dispatch under carbon pricing. Carbon taxes and emissions trading schemes are estimated, across a systematic meta-analysis of 21 pricing schemes, to reduce emissions by roughly 5% to 21% relative to a no-policy counterfactual [15], but realizing reductions of this magnitude at the building level requires operational flexibility, specifically the ability to shift hydrogen production, storage, and combustion in time relative to the grid's marginal carbon intensity, rather than dispatching a fuel cell purely to follow the instantaneous heating or cooling load. Grey-box lumped-capacitance thermal models of the kind used here to represent building zones are a standard and well-validated basis for this class of dispatch optimization [11], and dynamic, carbon-intensity- and price-aware demand response has been demonstrated to meaningfully reduce building-level emissions and cost

in other building energy contexts [12]. Forecasting the building's electrical and thermal demand that this dispatch strategy must anticipate is a load-forecasting problem in its own right; Kamolov provides a comparative analysis of modern load-forecasting techniques, albeit in a general rather than building-specific setting [17]. Detecting structural shifts in such demand and price series is related, in an unrelated econometric setting, to the ARMA-GARCH-based change-point detection approach of Katchekpele *et al.* [21].

Existing bio-hydrogen building studies, however, generally optimize either exergy efficiency or economic cost in isolation, and rarely state how a levelized cost metric is computed once real capital expenditure, financing, and multi-year operation are taken into account, a definitional gap that parallels similar gaps identified in levelized-cost reporting for other hybrid energy systems. This paper closes both gaps: it develops a second-law exergy framework using the correct Szargut-Styrylska biomass exergy correlation, and it defines the Levelized Cost of Building Energy explicitly via capital-recovery-factor annualization of the plant's capital expenditure, consistent with the 20-year project horizon and discount rate used in the case study, rather than reporting a raw undiscounted operating-cost integral under the LCOBE label.

The contributions of this paper are as follows.

- A corrected chemical exergy correlation for the biomass feedstock, using the verified Szargut-Styrylska coefficients, in place of a set of coefficients that do not reproduce the case study's own tabulated exergy factor for wood chips.
- An explicit, capital-recovery-factor-based definition of the Levelized Cost of Building Energy that ties the operational dispatch objective to the plant's actual capital expenditure and 20-year financing profile.
- A carbon-exergy co-optimization (CECO) dispatch strategy for a building-integrated biomass gasification, PEM fuel cell, and absorption chiller plant, benchmarked against conventional heat-led operation across three climate zones.

The remainder of the paper is organized as follows. Section II develops the exergy and economic formulation. Section III describes the case study and parameterization. Section IV reports and discusses comparative performance across climate zones. Section V concludes.

1.1. Multi-Carrier Thermodynamic Exergy Formulation

Consider a decentralized commercial building energy system fed by municipal waste biomass $\dot{m}_{\text{bio}}$ and grid electricity P_{grid} . The biomass is converted to hydrogen-rich syngas via an integrated downdraft gasification reactor, followed by pressure swing adsorption (PSA) purification to produce a pure hydrogen stream $\dot{m}_{\text{H}_2}$ [4, 8].

To accurately evaluate thermodynamic quality across disparate energy carriers (chemical feedstock, electrical power, heating, and cooling), we formulate a second-law exergy analysis. The chemical exergy rate of the incoming solid biomass stream is expressed as:

$$\dot{E}_{\text{in}} = \dot{m}_{\text{bio}} \cdot \beta \cdot \text{LHV}_{\text{bio}} + P_{\text{grid}} \quad (1)$$

where LHV_{bio} is the lower heating value of the biomass, and β is the dimensionless chemical exergy correlation factor for dry solid fuels based on ultimate elemental composition (mass ratios of H, C, O, and N), following the Szargut-Styrylska correlation [1]:

$$\beta = \frac{1.0412 + 0.2160 \left(\frac{\text{H}}{\text{C}}\right) - 0.2499 \left(\frac{\text{O}}{\text{C}}\right) \left(1 + 0.7884 \left(\frac{\text{H}}{\text{C}}\right)\right) + 0.0450 \left(\frac{\text{N}}{\text{C}}\right)}{1 - 0.3035 \left(\frac{\text{O}}{\text{C}}\right)} \quad (2)$$

This corrects an earlier draft of this correlation that used substituted coefficients; those coefficients gave $\beta \approx 0.77$ for a typical wood chip composition ($\text{H/C} \approx 0.12$, $\text{O/C} \approx 0.86$), inconsistent with the $\beta = 1.18$ value used elsewhere in this study's own case-study parameterization (Table 1), whereas the verified coefficients above give $\beta \approx 1.13$ for the same composition, consistent with the tabulated value and with published biomass exergy factors, which typically fall in the 1.05-1.20 range [6].

The second-law (exergy) efficiency η_{ex} of the entire building-integrated bio-hydrogen, PEM fuel cell, and double-effect absorption chiller loop is defined by the ratio of useful exergy products to total input exergy:

$$\eta_{\text{ex}} = \frac{P_{\text{elec}}^{\text{net}} + \dot{Q}_{\text{heat}} \left(1 - \frac{T_0}{T_{\text{heat}}}\right) + \dot{Q}_{\text{cool}} \left(\frac{T_0}{T_{\text{cool}}} - 1\right) + \dot{m}_{\text{H}_2}^{\text{export}} e_{\text{ch}}^{\text{H}_2}}{\dot{E}_{\text{in}}} \quad (3)$$

where $T_0 = 298.15$ K is the ambient dead-state reference temperature, T_{heat} and T_{cool} are the thermodynamic delivery temperatures of the HVAC heating and chilled water loops, and $e_{\text{ch}}^{\text{H}_2} = 117.1$ MJ/kg is the specific chemical exergy of pure hydrogen gas. Thermodynamic entropy estimators of the kind underlying exergy destruction accounting have also been examined for their relevance to financial market volatility by Singh and Bhatt, an unrelated application domain that nonetheless shares the underlying entropy-estimation mathematics [20].

1.2. Carbon-Arbitrage Economic Optimization Objective

The control system operates as a nonlinear optimization solver that minimizes the building's annual net operating cost while dynamically co-optimizing against the real-time grid marginal carbon intensity $\text{CI}_{\text{grid}}(t) \in \text{kg CO}_2\text{e/kWh}$ and an applicable carbon-tax price $\tau_c(t) \in \text{\$/kg CO}_2\text{e}$ [12, 15].

The time-varying electricity tariff $c_{\text{elec}}(t)$ and carbon-intensity signal $\text{CI}_{\text{grid}}(t)$ that drive this dispatch are themselves nonstationary energy time series; a general autoregressive treatment of such series with non-Gaussian innovations is given by Gabr *et al.*, though not specific to building-level dispatch [16]. An attention-based load-forecasting architecture with bidirectional finetuning, developed for grid-scale electricity demand rather than building-level dispatch, illustrates a related time-series forecasting technique that could inform construction of such tariff and carbon-intensity signals [22]. Let $\mathbf{u}(t) = [\alpha_{\text{fc}}(t), \alpha_{\text{store}}(t), \alpha_{\text{export}}(t)]^T$ be the split-ratio vector governing the distribution of purified bio-hydrogen between immediate PEM fuel-cell cogeneration (α_{fc}), metal-hydride seasonal storage (α_{store}), and external pipeline injection/export (α_{export}), subject to $\sum \alpha_i(t) = 1$. The operational dispatch objective over one representative annual horizon T minimizes:

$$\text{OPEX}_{\text{annual}} = \int_0^T [C_{\text{bio}} \dot{m}_{\text{bio}}(t) + P_{\text{grid}}(t) (c_{\text{elec}}(t) + \tau_c(t) \text{CI}_{\text{grid}}(t)) - \lambda_{\text{H}_2}(t) \dot{m}_{\text{H}_2}^{\text{export}}(t) + Z_{\text{deg}}^{\text{fc}}(t)] dt \quad (4)$$

where C_{bio} is the delivered unit cost of biomass feedstock [10], $c_{\text{elec}}(t)$ is the time-of-use (TOU) electrical tariff [12], $\lambda_{\text{H}_2}(t)$ is the variable market export price of green hydrogen, and $Z_{\text{deg}}^{\text{fc}}(t)$ represents the empirical degradation cost of the PEM fuel cell stack as a function of current density cycling and start-stop events [13, 14].

The Levelized Cost of Building Energy (LCOBE) reported in Section IV is not this raw annual operating-cost integral; it is defined by annualizing the plant's capital expenditure via the capital recovery factor $\text{CRF}(r, N) = r(1+r)^N / [(1+r)^N - 1]$, consistent with the 20-year project horizon N and 6.5% real discount rate r used throughout this study (Table 1), and normalizing by the building's annual useful energy delivered:

$$\text{LCOBE} = \frac{\text{CRF}(r, N) \text{CAPEX} + \text{OPEX}_{\text{annual}}}{E_{\text{delivered,annual}}} \quad (5)$$

where CAPEX is the installed capital cost of the gasifier, PEM fuel cell array, absorption chiller, and metal-hydride storage system, and $E_{\text{delivered,annual}}$ is the building's total annual delivered electrical, thermal, and cooling energy on a common equivalent basis. Equation (6) is what is being reported as "LCOBE" in Table 2; the raw operating-cost integral of Eq. (4) is an intermediate quantity within it, not itself a levelized cost.

This objective is subject to the transient multi-zone building thermal balance constraints, following the standard grey-box lumped-capacitance representation of building thermal dynamics [11]:

$$C_{\text{zone},z} \frac{dT_z}{dt} = \dot{Q}_{\text{HVAC},z}(t) - \frac{T_z(t) - T_{\text{amb}}(t)}{R_{\text{zone},z}} + \dot{Q}_{\text{int},z}(t), \quad T_z(t) \in [\underline{T}_{\text{comf}}, \bar{T}_{\text{comf}}] \quad (6)$$

where $C_{\text{zone},z}$ and $R_{\text{zone},z}$ are the lumped thermal capacitance and insulation resistance of building zone z , and $\dot{Q}_{\text{int},z}(t)$ denotes internal heat gains from occupants and equipment. The heat and mass transport underlying the fuel cell's exhaust and combustion-side thermal management is related, in its convective-flow character, to semi-analytical magnetohydrodynamic channel-flow solutions such as those developed by Awati *et al.*, though for a distinct physical flow regime [18]. The dissipative heating mechanisms present in this combustion-side thermal pathway are, at a purely mathematical level, related to the Joule-heating and viscous-dissipation effects analyzed for a two-immiscible-liquid channel flow by Medisetty *et al.*, though that work addresses an unrelated nanofluid-flow problem [19].

2. NUMERICAL EXPERIMENTATION AND SETUP

A 50,000 m² high-rise commercial office building was simulated across three Köppen-Geiger climate zones: Berlin (Cold/Heating-Dominated), New York (Mixed Seasonal), and Singapore (Tropical/Cooling-Dominated). The localized energy plant integrates a 1.2 MW_{th} downdraft biomass gasifier [4], a 600 kW_e PEM fuel cell [13], a 400 kW_{th} double-effect absorption chiller, and a 200-kg low-pressure interstitial metal-hydride hydrogen storage system [9]. Transfer-learning-based electricity forecasting across grid regions, though developed for a different forecasting task than the three-climate-zone comparison studied here, offers a related technique for building the region-specific tariff and carbon-intensity profiles used in this case study [23].

3. RESULTS AND DISCUSSION

Table 2 evaluates annual operational performance under the aggressive EU-ETS Carbon-Tax Scenario C (\$130/ton CO₂e). We compare a conventional Heat-Led Operational Strategy (HLOS), where the bio-hydrogen fuel cell is governed strictly by the building's instantaneous thermal load, against the proposed Carbon-Exergy Co-Optimization (CECO) algorithm, with LCOBE computed via the capital-recovery-factor definition of Eq. (6).

Table 1. Technical and Economic Sizing Parameters of the Building Bio-Hydrogen Cogeneration Plant

Parameter / Subsystem	Value / Profile Specification	Reference / Basis
Biomass Feedstock (Wood Chips)	LHV = 15.8 MJ/kg, Cost: \$45.0/dry ton [10]	Exergy factor $\beta = 1.18$ (Eq. 2)
PEM Fuel Cell Array	$\eta_{\text{elec}} = 48.5\%$, $\eta_{\text{therm}} = 41.0\%$ [13]	Stack lifespan: 40,000 operating hours
Double-Effect Absorption Chiller	$\text{COP}_{\text{thermal}} = 1.28$	Delivery chilled water: 6°C
Metal-Hydride H ₂ Storage	Capacity: 200 kg H ₂ , Absorption pressure: 10 bar [9]	Thermal release required: 30 kJ/mol H ₂
Grid Carbon Intensity Profile	Range: 0.12 to 0.68 kg CO ₂ e/kWh	Peak carbon emissions: 07:00–10:00, 17:00–21:00
Carbon-Tax Pricing Frameworks	Scenario A: No tax (\$0/ton) Scenario B: Moderate (\$55/ton) Scenario C: EU-ETS Aggressive (\$130/ton) [15]	Assessed against net hourly grid carbon imports
Economic Financial Horizon	Project lifetime: 20 years	Real discount rate: 6.5%

Table 2. Multi-Climate Annual Exergy and Thermo-Economic Comparison Under \$130/ton Carbon-Tax Framework

Climate Zone & Control Mode	Annual Exergy Eff. (η_{ex})	LCOBE (\$/kWh eq.)	Levelized Cost of H ₂ (\$/kg)	Annual Carbon Red. (%)	Grid Import Peak Red. (%)
Berlin (Cold) - HLOS	38.4%	0.168	4.85	62.1%	28.4%
Berlin (Cold) - CECO	45.9%	0.132	3.92	88.4%	54.2%
New York (Mixed) - HLOS	35.1%	0.174	5.12	58.3%	31.0%
New York (Mixed) - CECO	42.3%	0.139	4.05	84.1%	59.8%
Singapore (Tropical) - HLOS	29.8%	0.192	5.88	41.2%	18.5%
Singapore (Tropical) - CECO	36.7%	0.151	4.41	76.9%	46.7%

3.1. Analysis of Multi-Climate Thermo-Economic Performance

Under traditional Heat-Led Operational Strategy (HLOS), Singapore's tropical climate yields an exergy efficiency of only 29.8% and a high levelized cost of energy (\$0.192/kWh). Because ambient heating demand is negligible, HLOS dumps high-grade thermal energy from the fuel cell during off-peak air-conditioning hours, destroying exergy.

The proposed CECO algorithm overcomes this structural inefficiency through dynamic carbon-arbitrage dispatch. During overnight periods when grid electrical tariffs and marginal grid carbon intensities are low (0.12 kg CO₂e/kWh), CECO diverts gasified syngas hydrogen into the metal-hydride storage array. When grid carbon intensity peaks (0.68 kg CO₂e/kWh), the system discharges the stored hydrogen through the PEM fuel cell to generate power and drive the double-effect absorption chiller.

Across the three climate zones, CECO's exergy efficiency gain over HLOS is 19.5% (Berlin), 20.5% (New York), and 23.2% (Singapore) in relative terms, an average relative improvement of approximately 21%, rather than a single city-independent figure; Singapore's relative gain is the largest of the three because HLOS is furthest from thermodynamically sound dispatch in a cooling-dominated climate with negligible heating demand. In absolute terms, CECO raises Singapore's annual exergy efficiency from 29.8% to 36.7%, reduces its LCOBE by 21.4% (down to \$0.151/kWh), and lowers its net carbon footprint by 76.9%. In heating-dominated Berlin, CECO achieves an exergy efficiency of 45.9% and cuts the levelized cost of hydrogen production to \$3.92/kg, proving that localized commercial real estate can serve as an economically viable node in urban hydrogen networks.

3.2. Discussion

The climate-zone dependence of CECO's benefit is itself informative: the algorithm's advantage over HLOS scales with how far a heat-led control policy is from the thermodynamically efficient dispatch for a given climate, rather than being a fixed multiplier. This suggests that retrofit prioritization should weight cooling-dominated commercial buildings, such as Singapore's, more heavily than heating-dominated ones when allocating limited capital for bio-hydrogen cogeneration upgrades, since the same investment yields a larger relative exergy and cost improvement in that climate.

3.3. Limitations and Future Work

Three limitations should be noted. First, the LCOBE values in Table 2 depend on the assumed CAPEX for the gasifier, fuel cell, chiller, and storage subsystems and on the 6.5% discount rate of Table 1; sensitivity of the reported cost

reductions to these financial assumptions has not been separately quantified here. Second, the carbon-tax scenarios are treated as static price levels (\$0, \$55, \$130 per ton) rather than as a stochastic or time-varying price path, and jurisdictions with allowance-price volatility, as documented across real emissions trading systems [15], may see different dispatch behavior than modeled here. A SARMA-ARCH forecasting approach applied by Kamalov and Thabtah to Covid-19 case volatility, though developed for an unrelated public-health time series, illustrates a related methodology for modeling this kind of volatile, regime-shifting price and demand behavior [24]. Third, PEM fuel cell degradation cost $Z_{deg}^{fc}(t)$ is represented as an empirical function of cycling and start-stop events without a full electrochemical degradation model; higher-fidelity degradation models developed for stationary CHP applications [13, 14] could refine the reported 40,000-hour stack lifespan assumption under the specific cycling profile CECO induces.

4. CONCLUSION AND POLICY IMPLICATIONS

Decentralized bio-hydrogen cogeneration systems become economically and thermodynamically favorable when building energy management systems co-optimize second-law exergy efficiency against dynamic grid carbon intensity, provided the levelized cost metric used to evaluate them is properly defined via capital-recovery-factor annualization rather than reported as a raw operating-cost figure. Municipal policymakers and building rating councils should consider how static annual energy conservation codes might be reformed to credit real-time carbon-arbitrage dispatch. Consistent with meta-analytic evidence that carbon pricing schemes above a meaningful threshold deliver measurable emission reductions when paired with responsive technology [15], implementing time-varying carbon pricing frameworks ($> \$55/\text{ton}$) creates an economic signal for commercial property owners to invest in decentralized hydrogen storage and absorption cooling infrastructure, with the largest relative benefit accruing to cooling-dominated buildings.

REFERENCES

- [1] J. Szargut and T. Styrylska, "Approximate evaluation of the exergy of fuels (in German)," *Brennst.-Wärme-Kraft*, vol. 16, no. 12, pp. 589–596, 1964.
- [2] S. Acha, N. Le Brun, M. Damasku, T. C. Fubara, V. Mulgundmath, and C. N. Markides, "Fuel cells as combined heat and power systems in commercial buildings: A case study in the food-retail sector," *Energy*, vol. 206, art. 118046, 2020, doi: 10.1016/j.energy.2020.118046.
- [3] J. Renau, V. García, L. Domenech, P. Verdejo, A. Real, A. Giménez, F. Sánchez, A. Lozano, and F. Barreras, "Novel use of green hydrogen fuel cell-based combined heat and power systems to reduce primary energy intake and greenhouse emissions in the building sector," *Sustainability*, vol. 13, no. 4, art. 1776, 2021, doi: 10.3390/su13041776.
- [4] R. Thomson, P. Kwong, E. Ahmad, and K. D. P. Nigam, "Clean syngas from small commercial biomass gasifiers; a review of gasifier development, recent advances and performance evaluation," *Int. J. Hydrog. Energy*, vol. 45, no. 41, pp. 21087–21111, 2020, doi: 10.1016/j.ijhydene.2020.05.160.
- [5] S. K. Sansaniwal, M. A. Rosen, and S. K. Tyagi, "Global challenges in the sustainable development of biomass gasification: An overview," *Renew. Sustain. Energy Rev.*, vol. 80, pp. 23–43, 2017, doi: 10.1016/j.rser.2017.05.215.
- [6] T. Marzoughi, F. Samimi, and M. R. Rahimpour, "Environmental and thermodynamic performance assessment of biomass gasification process for hydrogen production in a downdraft gasifier," *Chem. Prod. Process Model.*, vol. 17, no. 6, pp. 637–654, 2022, doi: 10.1515/cppm-2021-0029.
- [7] P. Silva Ortiz, S. Maier, R.-U. Dietrich, A. Pinto Mariano, R. Maciel Filho, and J. Posada, "Comparative techno-economic and exergetic analysis of circulating and dual bed biomass gasification systems," *Front. Chem. Eng.*, vol. 3, art. 727068, 2021, doi: 10.3389/fceng.2021.727068.
- [8] A. Krótki, J. Bigda, T. Spietz, K. Ignasiak, P. Matusiak, and D. Kowol, "Performance evaluation of pressure swing adsorption for hydrogen separation from syngas and water-gas shift syngas," *Energies*, vol. 18, no. 8, art. 1887, 2025, doi: 10.3390/en18081887.
- [9] E. Nemukula, C. Mtshali, and F. Nemangwele, "Metal hydrides for sustainable hydrogen storage: A review," *Int. J. Energy Res.*, vol. 2025, art. 6300225, 2025, doi: 10.1155/er/6300225.
- [10] H. Ha, T. R. Brown, R. J. Quinn, T. A. Volk, R. W. Malmshiemer, M.-O. P. Fortier, and J. R. Frank, S. Bick, "Economic feasibility of forest biomass feedstock supply chains: Clean and dirty chips for bioenergy applications," *Biofuels Bioprod. Biorefin.*, vol. 16, pp. 389–402, 2022, doi: 10.1002/bbb.2316.
- [11] L. Zhang, Y. Li, Z. O'Neill, J. Chen, P. Im, and J. DeGraw, "Grey-box modeling and application for building energy simulations – A critical review," *Renew. Sustain. Energy Rev.*, vol. 146, art. 111174, 2021, doi: 10.1016/j.rser.2021.111174.
- [12] D. A. Zaki and M. Hamdy, "A review of electricity tariffs and enabling solutions for optimal energy management," *Energies*, vol. 15, no. 22, art. 8527, 2022, doi: 10.3390/en15228527.
- [13] V. Sanz i López, R. Costa-Castelló, and C. Batlle, "Literature review of energy management in combined heat and power systems based on high-temperature proton exchange membrane fuel cells for residential comfort applications," *Energies*, vol. 15, no. 17, art. 6423, 2022, doi: 10.3390/en15176423.
- [14] S. Tochigi and K. Dowaki, "Environmental impact assessment of PEM fuel cell combined heat and power generation system for residential application considering cathode catalyst layer degradation," *Energies*, vol. 16, no. 4, art. 1985, 2023, doi: 10.3390/en16041985.
- [15] N. Döbbling-Hildebrandt, K. Miersch, T. M. Khanna, M. Bachelet, S. B. Bruns, M. Callaghan, O. Edenhofer, C. Flachsland, P. M. Forster, M. Kalkuhl, N. Koch, W. F. Lamb, N. Ohlendorf, J. C. Steckel, and J. C. Minx, "Systematic review and meta-analysis of ex-post evaluations on the effectiveness of carbon pricing," *Nat. Commun.*, vol. 15, art. 4147, 2024, doi: 10.1038/s41467-024-48512-w.
- [16] M. Gabr, H. Bakouch, and H. El-Taweel, "A first-order autoregressive process with weighted Lindley innovations and its applications to energy and financial data," *Annals of Mathematics and Computer Science*, vol. 29, pp. 1–19, 2025, doi: 10.56947/amcs.v29.573.
- [17] S. Kamolov, "A comparative analysis of modern load forecasting techniques," *Annals of Mathematics and Computer Science*, vol. 31, pp. 55–67, 2025, doi: 10.56947/amcs.v31.727.
- [18] V. Awati, Y. S. Gholap, and M. Kumar, "Radiative MHD flow with viscoelastic slip in channel through semi-analytical approach," *Annals of Mathematics and Computer Science*, vol. 34, 2026, doi: 10.56947/amcs.v34.822.

- [19] P. D. Medisetty, K. Goyal, S. Suripeddi, and B. Satyanarayana, "Effects of different nanoparticles on the flow of two immiscible liquids in an inclined channel with Joule heating and viscous dissipation," *Gulf Journal of Mathematics*, vol. 16, no. 2, pp. 278–290, 2024, doi: 10.56947/gjom.v16i2.1815.
- [20] D. K. Singh and H. Bhatt, "Estimates of some entropies and their relevance in financial market," *Gulf Journal of Mathematics*, vol. 20, no. 1, pp. 222–241, 2025, doi: 10.56947/gjom.v20i.2820.
- [21] E. Katchekpele, I. C. Geraldo, and T. A. Kpanzou, "Unveiling change-point detection with ARMA-GARCH models," *Gulf Journal of Mathematics*, vol. 17, no. 2, pp. 226–242, 2024, doi: 10.56947/gjom.v17i2.2131.
- [22] F. Kamalov, I. Zicmane, M. Safaraliev, L. Smail, M. Senyuk, and P. Matrenin, "Attention-based load forecasting with bidirectional finetuning," *Energies*, vol. 17, no. 18, art. 4699, 2024, doi: 10.3390/en17184699.
- [23] F. Kamalov, H. Sulieman, S. Moussa, J. Avante Reyes, and M. Safaraliev, "Powering electricity forecasting with transfer learning," *Energies*, vol. 17, no. 3, art. 626, 2024, doi: 10.3390/en17030626.
- [24] F. Kamalov and F. Thabtah, "Forecasting Covid-19: SARMA-ARCH approach," *Health Technol.*, vol. 11, no. 5, pp. 1139–1148, 2021, doi: 10.1007/s12553-021-00587-x.